Representing spatial variability of timber material properties by means of hierarchical modeling

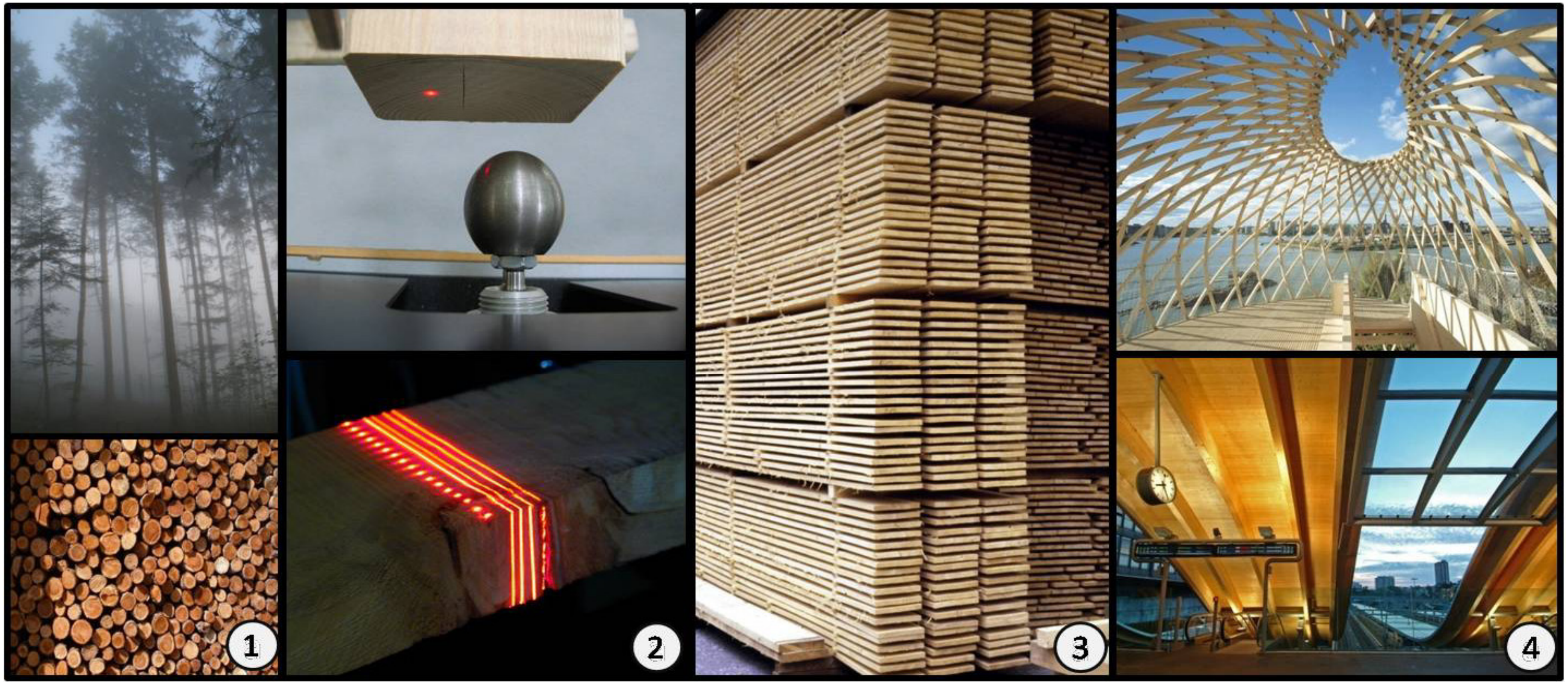
Markus K. Sandomeer

Jochen Köhler

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

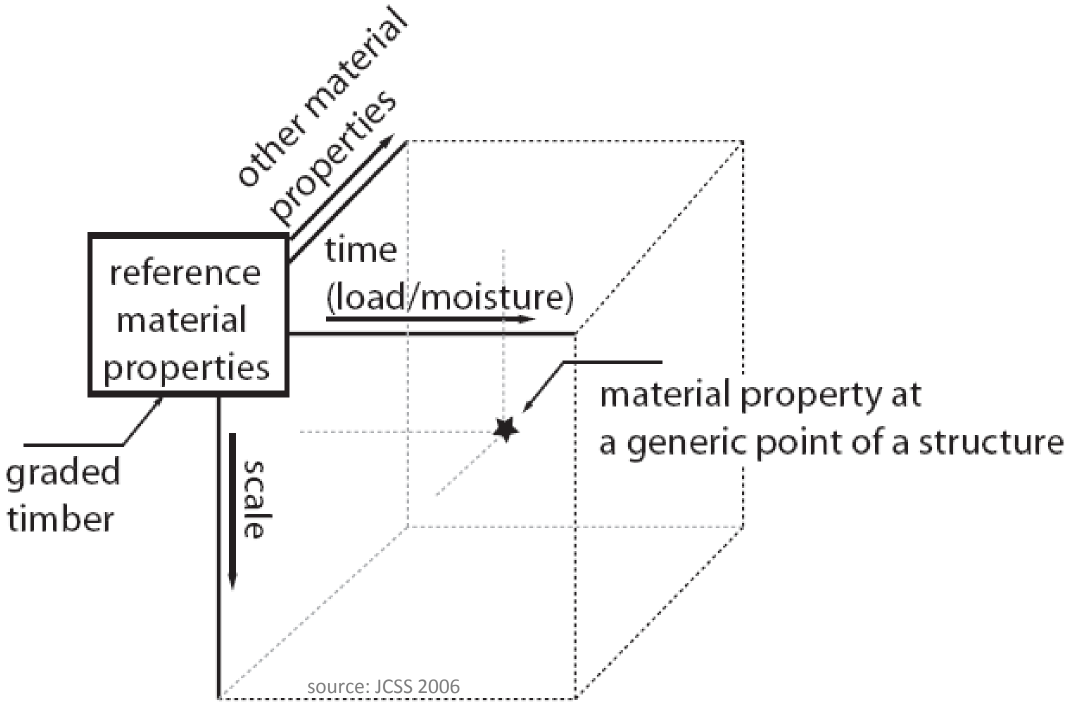
Materials Science & Technology

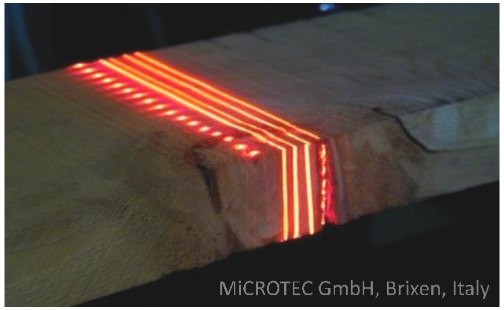
Swiss research project in close connection to
COST Action E53: „Quality Control of Wood and Wood Products“



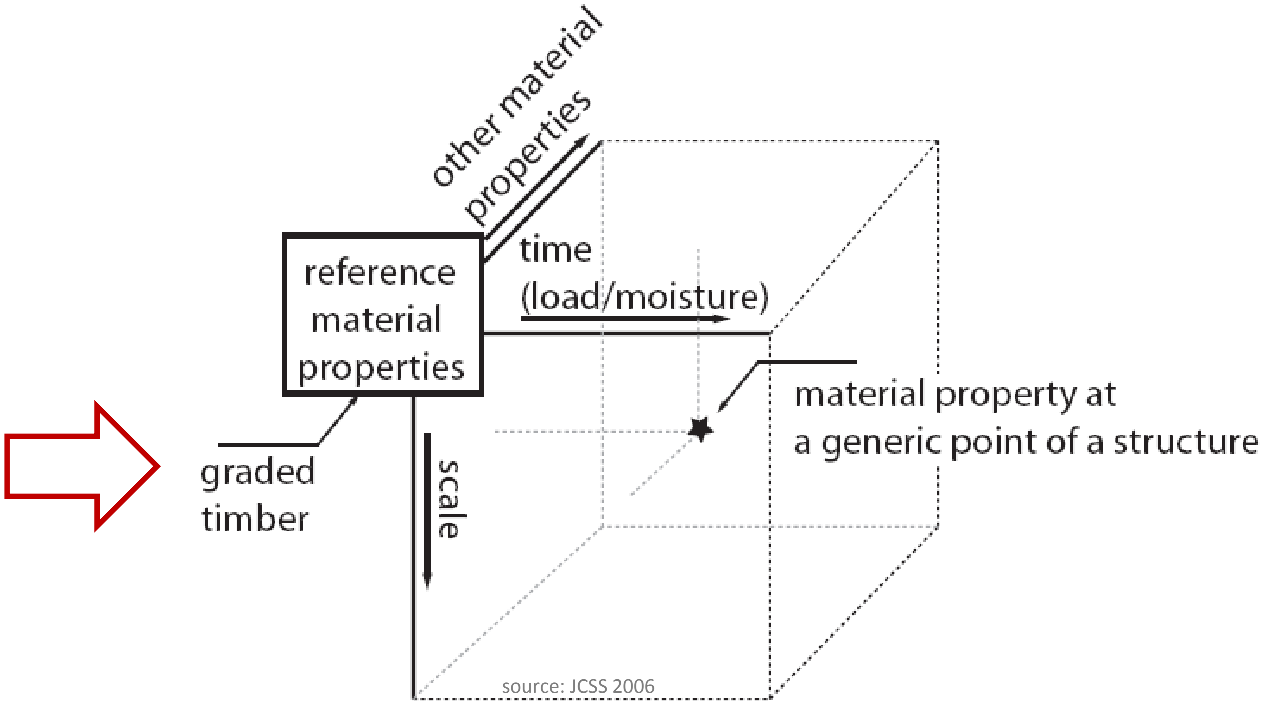
pictures 1, 3 and 4: www.lignum.ch; pictures 2: MiCROTEC GmbH, Brixen, Italy

motivation


other material properties
reference material properties
time (load/moisture)
material property at a generic point of a structure
graded timber
scale
source: JCSS 2006



CFPF, Le Mont-sur-Lausanne



OUTPUT of grading procedure → INPUT for modeling timber material properties

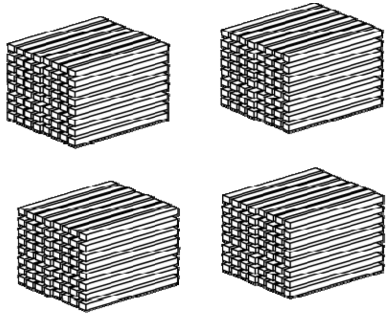
- aspects of hierarchical modeling
- example of application
- conclusions and outlook

aspects of hierarchical modeling

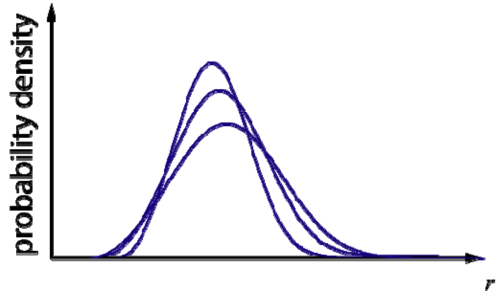
aspects of hierarchical modeling

overview

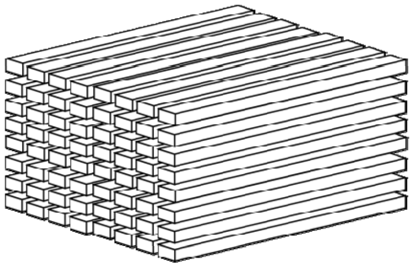
MACRO



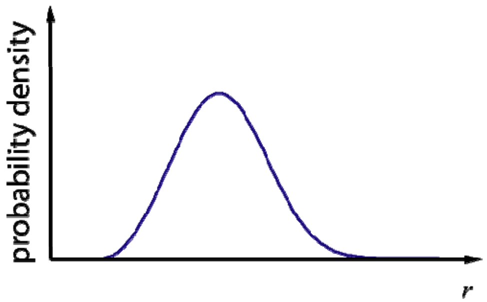
Sequence of Lots



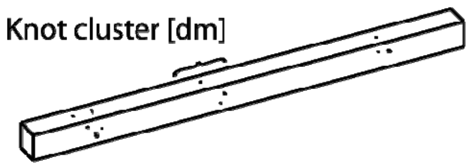
MESO



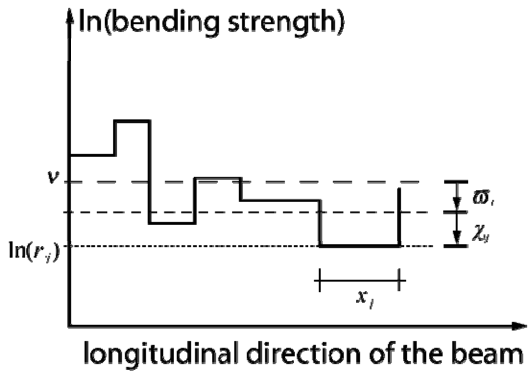
Lot of Structural Elements / Test Specimen



MICRO



Structural Element / Test Specimen [m]
=> sequence of weak zones



overview

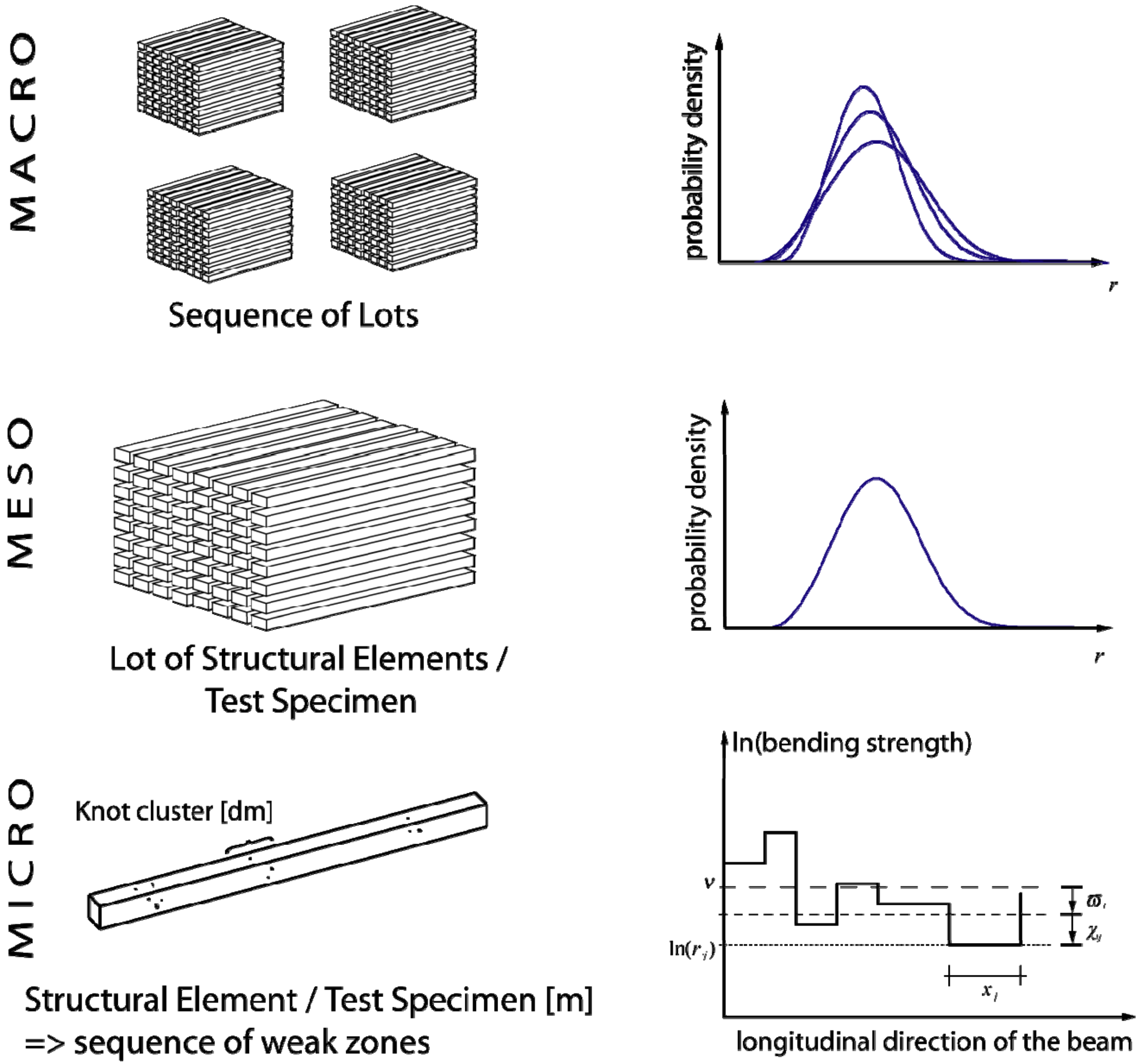
great advantage:
it is operational, for

probabilistic calculations

sampling

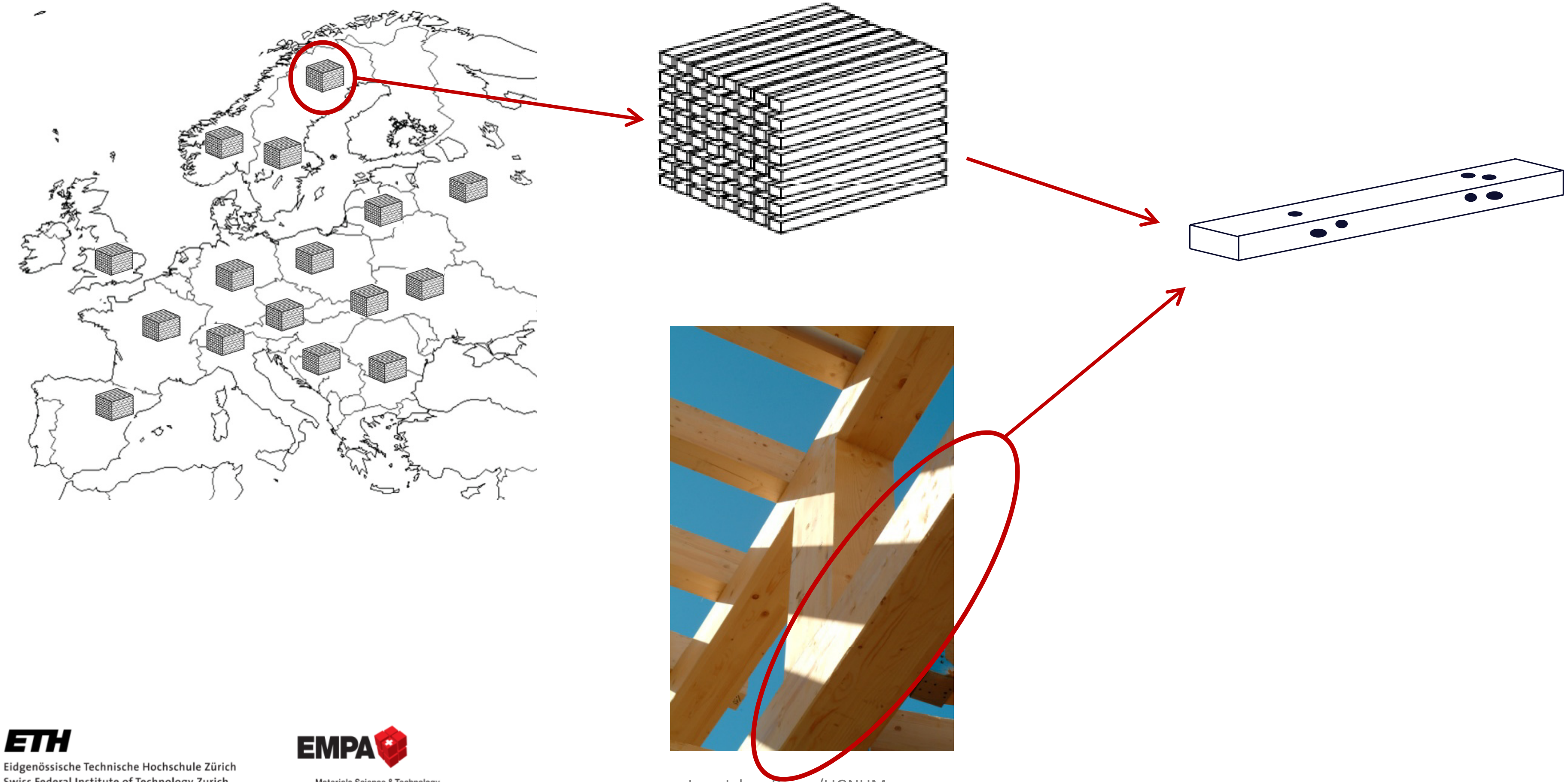
estimation

quality control



micro scale

one specimen/timber element



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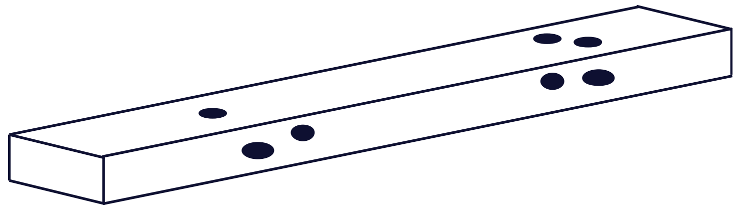
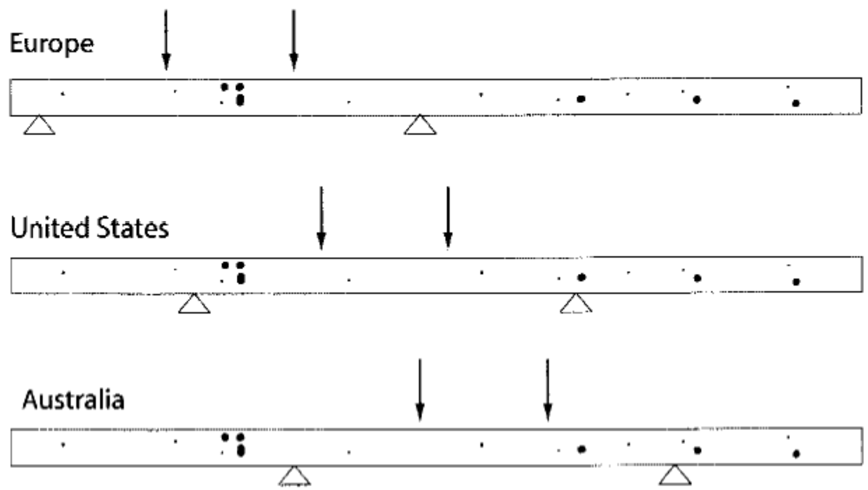
specimen/element



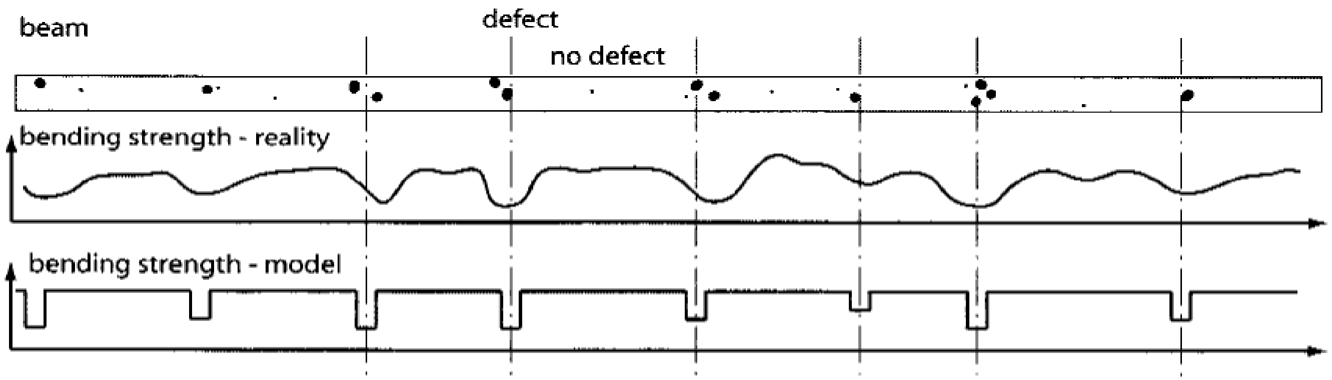
- variations and irregularities in the timber material itself
- modeling on the micro-scale includes physical considerations

influence of national test standards



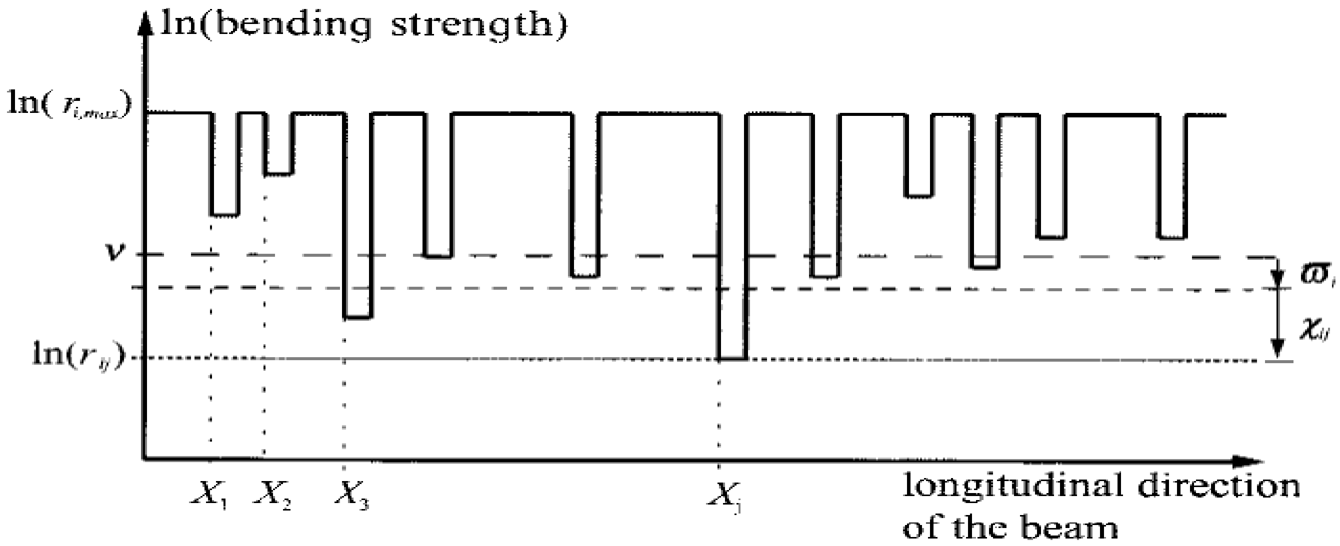
ideal brittle material model - e.g. Weibull 1939

series system of weak sections – Riberhold, Madsen 1979



Due to the discrete distribution of knots and knot clusters, an idealized model is proposed in terms of discrete weak sections separated by strong sections – sections of clear wood.

model for longitudinal variation - Isaksson 1999:

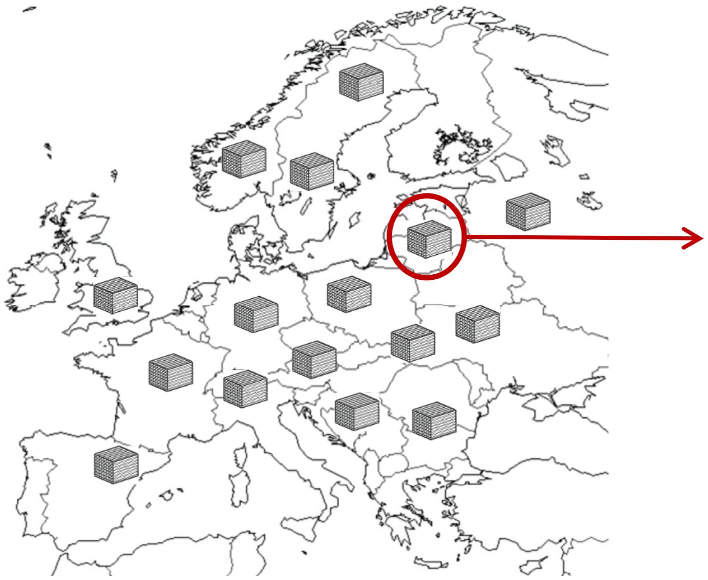


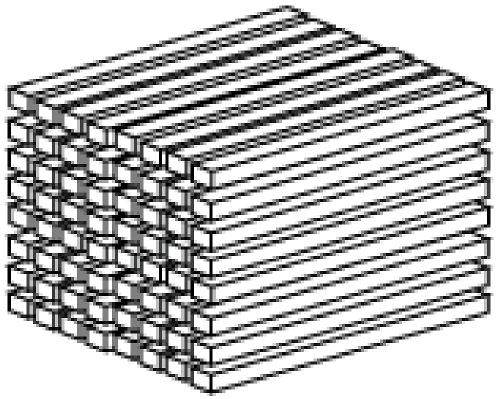
two level hierarchical model with two random variables:

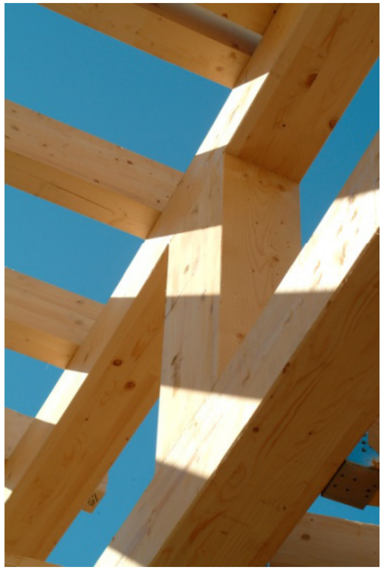
1. distribution of weak sections – modeled by Poisson process
2. distance between the weak sections – exponentially distributed

meso scale

one sub-population/sample/component
containing several elements







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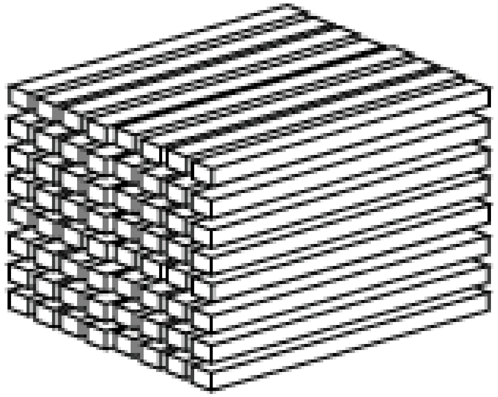
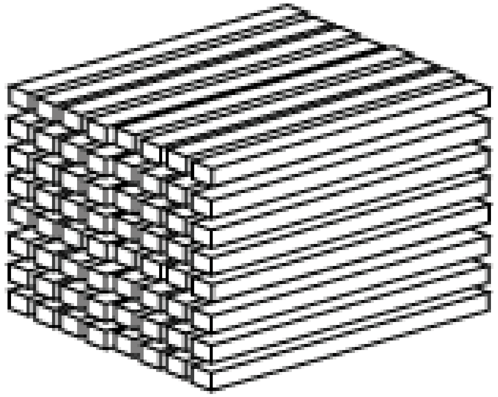
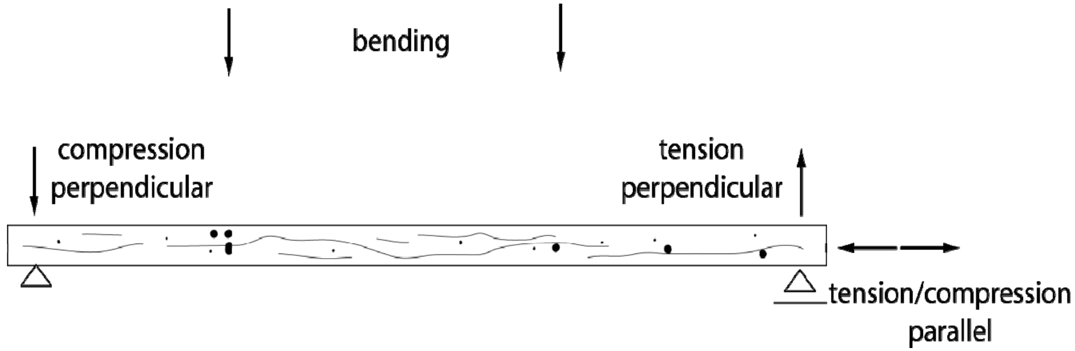
variation between timber test specimens or components within one sub-population



1. representative sample of timber specimens of one particular sub-population

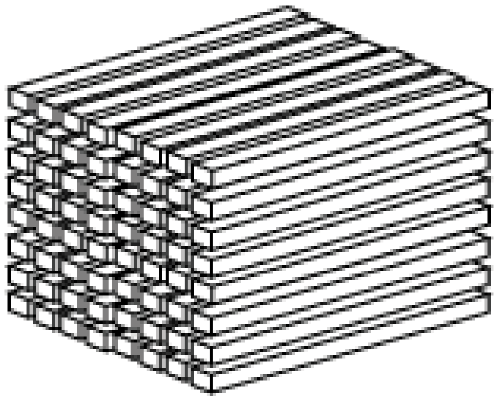
variation between timber test specimens or components within one sub-population

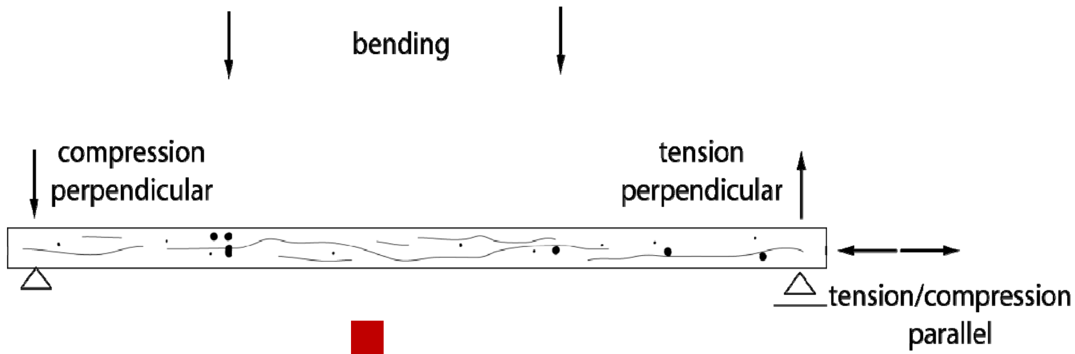




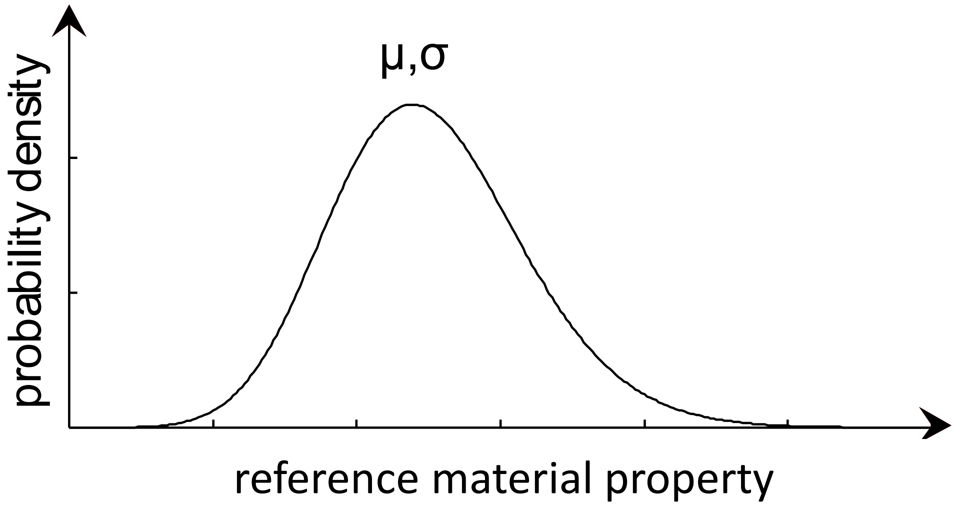
2. standard test specimen tested under standard testing conditions, e.g. according to EN 408

variation between timber test specimens or components within one sub-population





3. probability density function of the reference material property can be assessed; distribution parameters are estimated based on test results



Material properties are represented by random variables , statistical characteristics of these variables are described by distribution models which parameters are calibrated according to data taken from standard tests.

Guidance of the distribution type is given for instance in the probabilistic model code of JCSS:

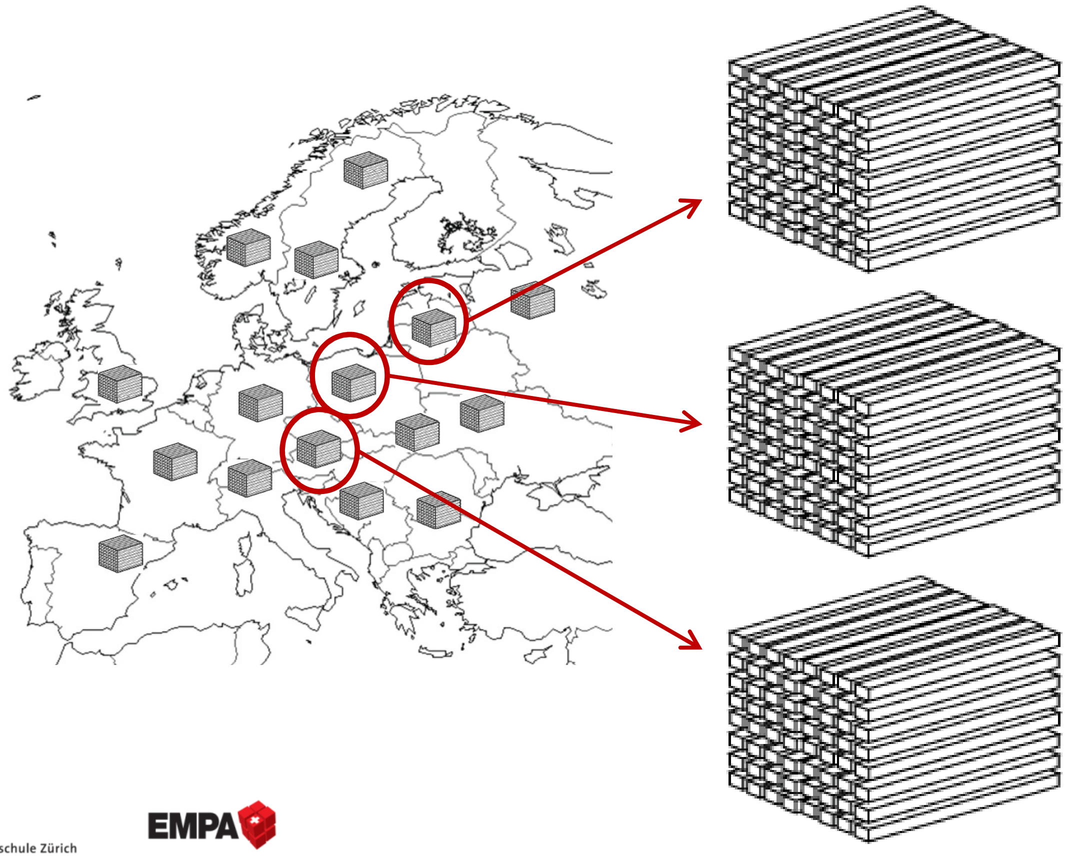
Table 2: Probabilistic models for reference properties for structural timber.

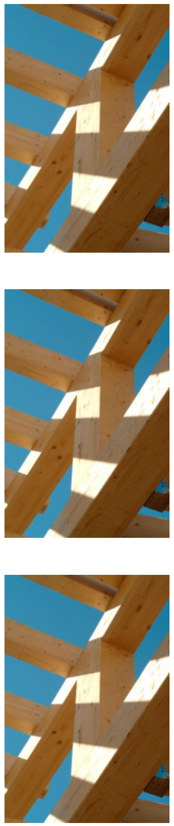
	Distribution	COV
Bending strength R_m	Lognormal	0.25
Bending MOE: E_m	Lognormal	0.13
Density ρ_{den}	Normal	0.1

JCSS probabilistic model code, 3.5: properties of timber; August, 2006

macro scale

different sub-populations/samples of different suppliers, origins, …





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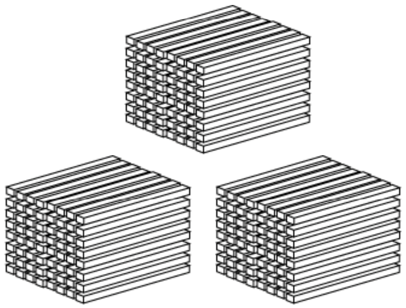
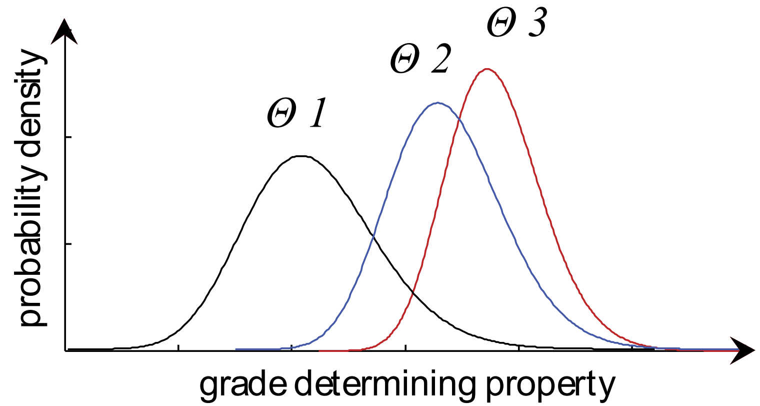
variation between sub-populations



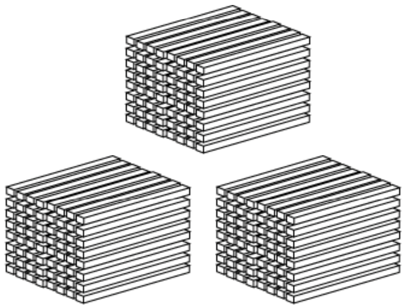


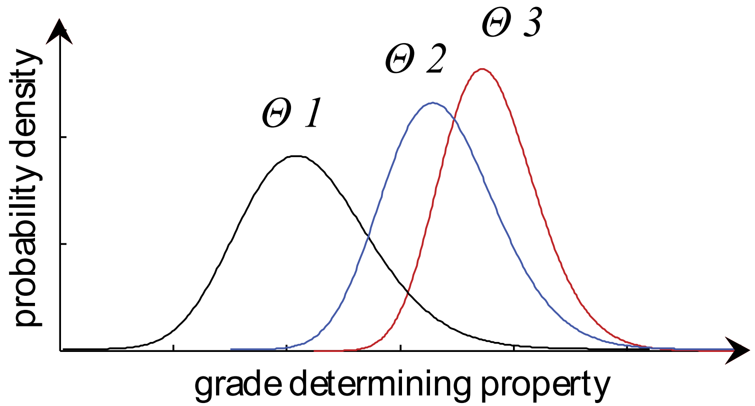
problem

Structural timber is utilized as graded material which has to fulfill the required characteristic values of the reference properties assigned to one specific strength-class.

When different sub-populations may be assigned to the same grade, the statistical properties might be different, although the characteristic values are similar.

variation between sub-populations





solution

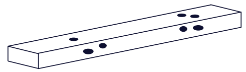
1. The sample moments of each sub-population are quantified and functional relationships between the sample moments are derived.
2. Alternatively, the macro variability can be explicitly assessed if the applied grading scheme can be formalized to a probabilistic framework which takes into account all uncertainties involved into the grading procedure.

example of application

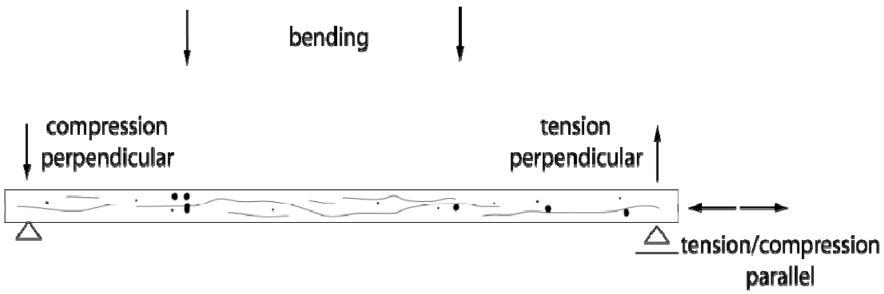
comparison of timber material properties
of different European origins

hierarchical structure – step 1

results of testing: samples with n individual tension strength measurements



MICRO

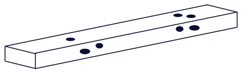


- test arrangement acc. to EN 408,
- assessment of characteristic values acc. to EN 384 for subdivision into strength classes acc. to EN 338 (meso scale)

timber density $x_i = (x_1, x_2, \ldots, x_n)^T$	timber bending strength and bending stiffness $z_i = (z_1, z_2, \ldots, z_n)^T$	raw data acc. to EN 408 EN 384
none	$x_i = \ln(z_i)$	transformation

hierarchical structure – step 2

results of testing: samples with n individual tension strength measurements



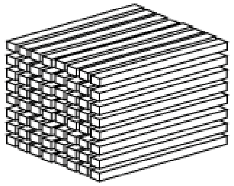
MICRO

sample mean and standard deviation

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

distribution parameters for normal distributed *ln*-values of tension strength

$$\mu = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma = \sqrt{\frac{n+1}{n}}\sqrt{\frac{v}{v-2}}s$$



with $v = n-1$

MESO

example

Central Europe

origin	μ	σ
CE_A	3.1417	0.4161
CE_B	3.1801	0.4597
CE_C	3.3000	0.3334
CE_D	3.2062	0.4094

Northern Europe

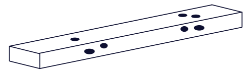
origin	μ	σ
NE_A	3.2520	0.3067
NE_B	3.1446	0.3429
NE_D	3.3778	0.3593
NE_E	3.2116	0.3382
NE_F	3.2694	0.2634
NE_G	3.4224	0.2930
NE_H	3.2718	0.3746

both

origin	μ	σ
CE_A	3.1417	0.4161
CE_B	3.1801	0.4597
CE_C	3.3000	0.3334
CE_D	3.2062	0.4094
NE_A	3.2520	0.3067
NE_B	3.1446	0.3429
NE_D	3.3778	0.3593
NE_E	3.2116	0.3382
NE_F	3.2694	0.2634
NE_G	3.4224	0.2930
NE_H	3.2718	0.3746

hierarchical structure – step 3

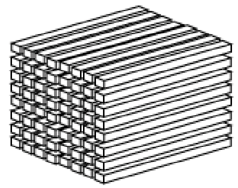
results of testing: samples with n individual tension strength measurements



MICRO

sample mean and standard deviation

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$



distribution parameters for normal distributed ln-values of tension strength

$$\mu = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma = \sqrt{\frac{n+1}{n}}\sqrt{\frac{v}{v-2}}\,s \qquad \text{with } v = n-1$$

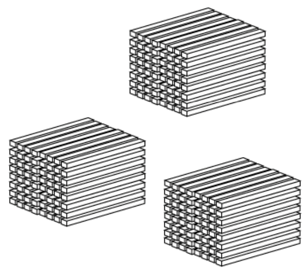
MESO

mean value of all µ
standard dev. of all µ

$$\mathrm{M}_\mu = \frac{1}{n}\sum_{i=1}^{n} \mu_i \qquad \Sigma_\mu = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\mu_i - \mathrm{M}_\mu\right)^2}$$

mean value of all σ
standard dev. of all σ

$$\mathrm{M}_\sigma = \frac{1}{n}\sum_{i=1}^{n} \sigma_i \qquad \Sigma_\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\sigma_i - \mathrm{M}_\sigma\right)^2}$$



MACRO

hierarchical structure – step 4

assessment of the bi-normal distribution parameters by means of the maximum likelihood method

bi-normal distribution:

$$f(\mu_i,\sigma_i) = \frac{1}{2\pi\Sigma_\mu\Sigma_\sigma\sqrt{1-\rho^2}} exp\left(-\frac{z}{2\left(1-\rho^2\right)}\right)$$

with

$$z = \frac{\left(\mu_i - M_\mu\right)^2}{\Sigma_\mu^2} - \frac{2\rho\left(\mu_i - M_\mu\right)\left(\sigma_i - M_\sigma\right)}{\Sigma_\mu\Sigma_\sigma} + \frac{\left(\sigma_i - M_\sigma\right)^2}{\Sigma_\sigma^2}$$

and

$$\rho = cor(\mu_i,\sigma_i)$$

Central Europe

origin	μ	σ
CE_A	3.1417	0.4161
CE_B	3.1801	0.4597
CE_C	3.3000	0.3334
CE_D	3.2062	0.4094

M	3.2070	0.4047
Σ	0.0674	0.0525
ρ	-0.8346	

Northern Europe

origin	μ	σ
NE_A	3.2520	0.3067
NE_B	3.1446	0.3429
NE_D	3.3778	0.3593
NE_E	3.2116	0.3382
NE_F	3.2694	0.2634
NE_G	3.4224	0.2930
NE_H	3.2718	0.3746

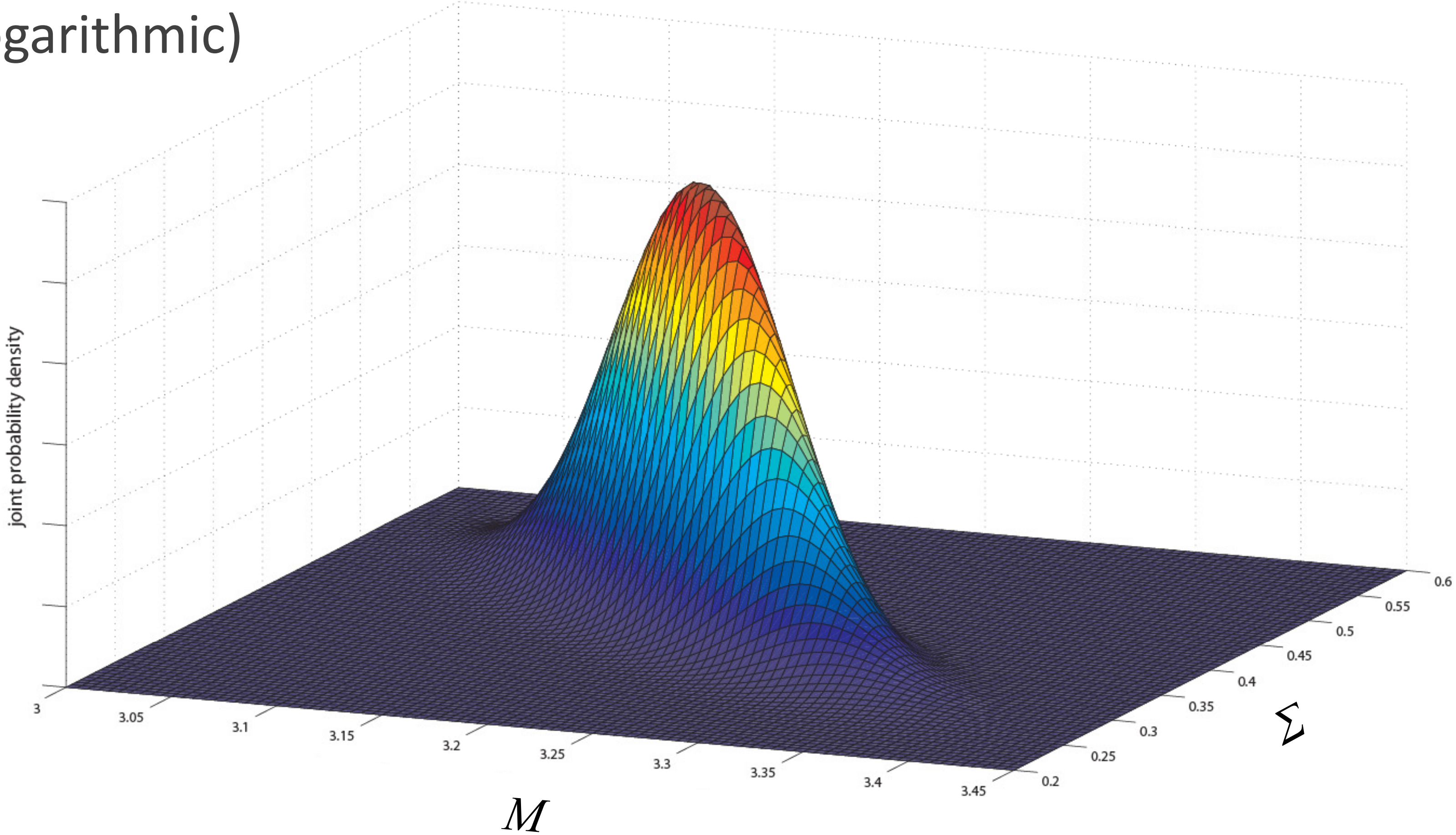
M	3.2785	0.3255
Σ	0.0947	0.0393
ρ	-0.1683	

both

origin	μ	σ
CE_A	3.1417	0.4161
CE_B	3.1801	0.4597
CE_C	3.3000	0.3334
CE_D	3.2062	0.4094
NE_A	3.2520	0.3067
NE_B	3.1446	0.3429
NE_D	3.3778	0.3593
NE_E	3.2116	0.3382
NE_F	3.2694	0.2634
NE_G	3.4224	0.2930
NE_H	3.2718	0.3746

M	3.2525	0.3543
Σ	0.0897	0.0579
ρ	-0.5207	

joint probability density function of macro scale mean and standard deviation (M,Σ) for Central European timber tension strength (logarithmic)



- Macro scale model still could be improved by using the natural conjugate prior of a normal distribution which is the Normal-Inverse-Gamma-2 distribution.

conclusions

conclusions

- Grading of structural timber has to ensure that the required reference material properties are fulfilled which are the basis for design.
- Concept of hierarchical modeling operational tool, not only for probabilistic calculations but also in sampling, estimation and quality control.
- Hierarchical modeling allows convenient examination of differences across populations.
- When comparing test results of different laboratories it is not necessary to transmit whole datasets; the distribution parameters are sufficient for further calculations.

Thank you for your attention!

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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sandomeer@ibk.baug.ethz.ch