A Probabilistic Approach for Robustness Evaluation of Timber Structures

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Aim of Research (WG 3 questions)

- How to model and assess reliability of timber structures modelled as systems ?
- Ductile / brittle failures?
- Key elements – how to design? To which reliability level?
- Robustness evaluation for timber structures?
- How is robustness requirements in Eurocodes handled for timber structures? Information in National Annexes?

Aim of Research

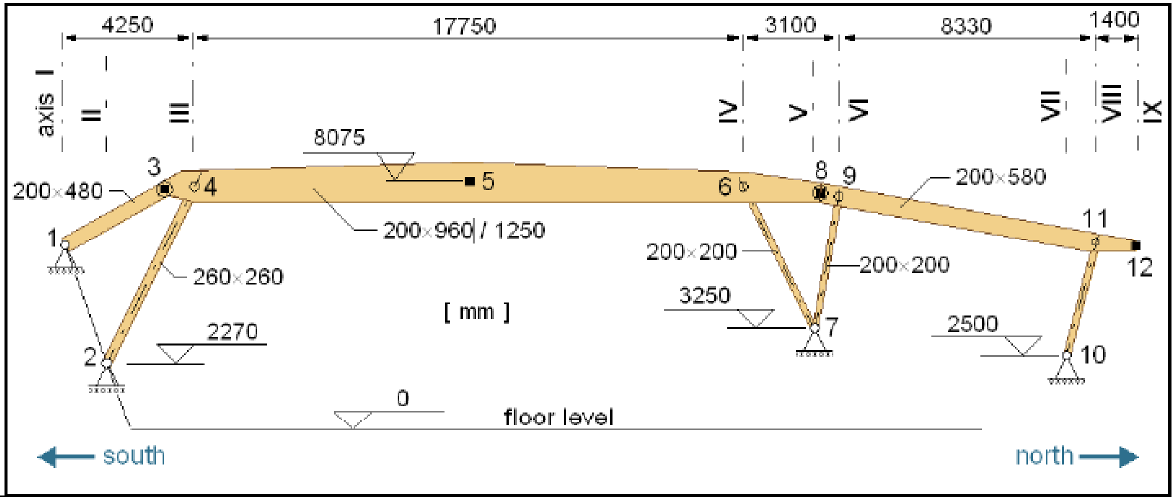
- Robustness evaluation of timber structure - Norwegian sports centre with a structural system consisting of 14 Glulam frames supporting the roof over the main court
- Robustness framework - Danish Code
- Probalbilistic modelling based on: JCSS – Probabilistic Model Code. Part 3: Resistance Models.

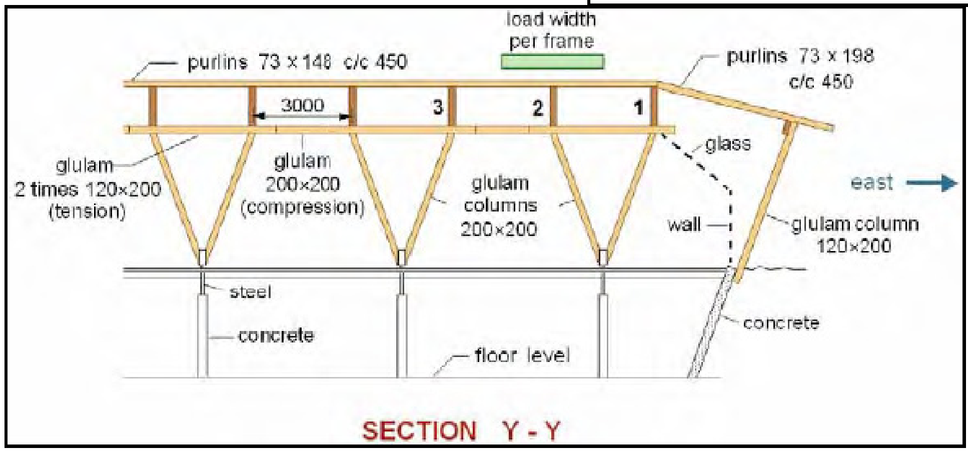
Content

- Introduction
- Robustness
- Probabilistic Model
  - -Failure mode
  - -Failure elements
  - -Stochastic model

- Reliability Analysis of the Glulam Frame
- Removal of Key Elements
- Conclusion

Introduction – Norwegian sports centre





Robustness

Siemens superarena Copenhagen,
Januar, 2003

Robustness - Levels

- Level I

$$I = \frac{Q_{Failure|D}}{Q_{Failure}}$$

- Level II

$$I = \frac{\beta_{Failure|D}}{\beta_{Failure}}$$

- Level III

$$I = \frac{R_{dir}}{(R_{dir} + R_{indir})}$$

Ellingwood, B.R. (2002) *Load and resistance factor criteria for progressive collapse design*.
Canisius, T.D.G. et al. (2007) *Robustness of structural systems – a new focus for the joint committee on structural safety (JCSS)*
Backer, J.W et al. (2007) *On the assessment of robustness*.

Robustness – Danish code DS 409 ?

Definition of robustness and key elements

- A structure is robust:
  - when those parts of the structure essential for the safety only have little sensitivity with respect to unintentional loads and defects,

    or
  - when extensive failure of the structure will not occur if a limited part of the structure fails.
- Key element:
  - limited part of structure, which has an essential importance for the robustness of the structure in the way that a possible failure of the key element implies a failure of the entire structure or significant parts of it.

Robustness – Danish code DS 409

Robustness: documented by a technical review where at least one of the following criteria is fulfilled:

a) demonstrate that those **parts of the structure essential for the safety only have little sensitivity** with respect to unintentional loads and defects

or

a) demonstrate a load case with '**removal of a limited part of the structure**' in order to document that an extensive failure of the structure will not occur if a limited part of the structure fails

or

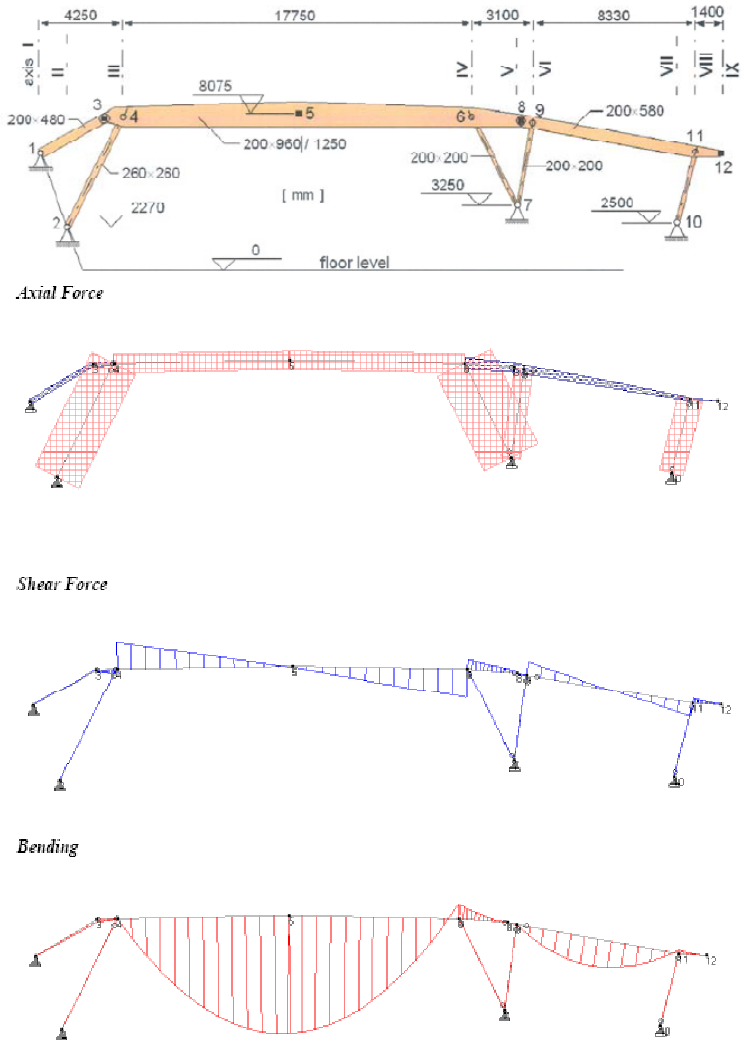
a) demonstrate sufficient safety of **key elements**, such that the entire structure with one or more key elements has the same reliability as a structure where robustness is documented by b).

Sørensen et al. (2006) *Danish Requirements to Robustness of Structures - Background and Implementation*

Probabilistic model - failure modes

1. Failure in column 2-4 (N)
2. Failure in column 6-7 (N)
3. Failure in column 7-9 (N)
4. Failure in column 10-11 (N)
5. Failure in the main beam at point 5 (N+M)
6. Failure in the main beam at point 6 (N+M)
7. Failure in beam 9-11 (M+N)
8. Failure in the main beam at point 4 (V)
9. Failure in the main beam at point 6 (V)
10. Failure due to a combination of tension perpendicular to grain and shear at point 5
11. Failure in the main beam at point 5 due to deflection.

(stress perpendicular to grain – due to moisture)
(connections)



Frühwald et al. (2007) *Design of safe timber structures – How can we learn from structural failures in concrete, steel and timber?*

Probabilistic model - failure elements (Brittle)

- Failure elements 1-4 - Failure in column (N)

$$g_i = 1 - \frac{N_S}{N_R} X_R = 1 - \frac{a_i G + b_i Q}{k_c A f_{c,0} k_{mod}} X_R \qquad i = 1,\dots,4$$

- Failure elements 5-7 - Failure in the main beam (N+M)

$$g_i = 1 - \left(\frac{N_S}{N_R} + k_m \frac{M_S}{M_R}\right) X_R = 1 - \left(\frac{a_i G + b_i Q}{k_c A f_{c,0} k_{mod}} + k_m \frac{c_i G + d_i Q}{W f_{m,a} k_{mod} k_h}\right) X_R \qquad i = 5,\dots,7$$

- Failure elements 8-9 - Failure in the main beam (V)

$$g_i = 1 - \frac{V_S}{V_R} X_R = 1 - \frac{e_i G + f_i Q}{\frac{2}{3} A f_v k_{mod}} X_R \qquad i = 8,\dots,9$$

- Failure elements 10 - Failure in the main beam – perpendicular stress

$$g_{10} = 1 - \left(\frac{V_S}{V_R} + \frac{M_S}{M_{R,90}}\right) X_R = 1 - \left(\frac{e_{10} G + f_{10} Q}{\frac{2}{3} A f_v k_{mod}} + \frac{(c_{10} G + d_{10} Q) k_p}{W k_{dis} k_{vol} f_{t,90} k_{mod} k_h}\right) X_R$$

- Failure elements 11 - deflection

$$g_{11} = 1 - \frac{w_{net,fin}}{\delta_L} X_R = 1 - \frac{e_{11} G (1 + k_{def}) + f_{11} Q}{\delta_L} X_R$$

Probabilistic model - stochastic model

Variable	Distribution	Expected value	COV	Designation
f_m	LN	49.9	0.15	Bending strength [13]
$f_{c,0}$	LN	$5\mu_{fm}^{0.45}$	$0.8V_{fm}$	Compression strength along grain [15]
f_v	LN	$0.2\mu_{fm}^{0.8}$	V_{fm}	Bending strength [15]
$f_{t,90}$	W	$0.0015\mu_\rho$	$2.5V_\rho$	Shear strength [15]
X_R	LN	1	0.05	Model uncertainty on short-term bearing capacity [15]
G	N	2.5 kN/m	0.1	Permanent load [13] (load width 3 m)
Q_g	G	3.13 kN/m	0.4	Variable load – snow [13] (load width 3 m)
A	N	1*	0.01	Area, *) multiplied with design value [18]
W	N	1*	0.01	Modulus,*) multiplied with design value [18]
k_c	N	1*	0.01	Instability,*) multiplied with design value [18]
C	D	0.8	-	Shape factor for snow [13]
k_h	D	1*	-	Size effect factor,*) multiplied with design value [17]
k_m	D	0.7	-	Re-distribution of stresses factor [17]
k_{dis}	D	1.4	-	Stress distribution factor in apex zone [17]
k_{vol}	D	1*	-	Volume factor in apex zone,*) multiplied with design value [17]
k_{mod}	D	0.9	-	Strength modification factor [15, 17]
k_{def}	D	1	-	Stiffness modification factor [15, 17]
k_p	D	0.007	-	Tensile stress perpendicular to the grain factor [17]
δ_L	D	0.089 mm	-	Deflection limit [17]

Table 1: Statistical characteristics (N:Normal, LN:Lognormal, G:Gumbel, W:2-pWeibull, D:Deterministic).

Reliability Analysis of the Glulam Frame

- Element reliability indices.

1	2	3	4	5	6	7	8	9	10	11
5.58	3.40	6.55	5.76	6.58	5.37	6.05	4.96	4.81	6.31	3.18

- Tentative target reliability indices β (and associated target failure rates) related to one year reference period and ultimate limit state

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences failure
Large (A)	$\beta = 3.1\ (P_f \approx 10^{-3})$	$\beta = 3.3\ (P_f \approx 5 \cdot 10^{-4})$	$\beta = 3.7\ (P_f \approx 10^{-4})$
Normal (B)	$\beta = 3.7\ (P_f \approx 10^{-4})$	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$
Small (C)	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$	$\beta = 4.7\ (P_f \approx 10^{-6})$

JCSS (2001) – *Probabilistic Model Code. Part1: Basic of design.*

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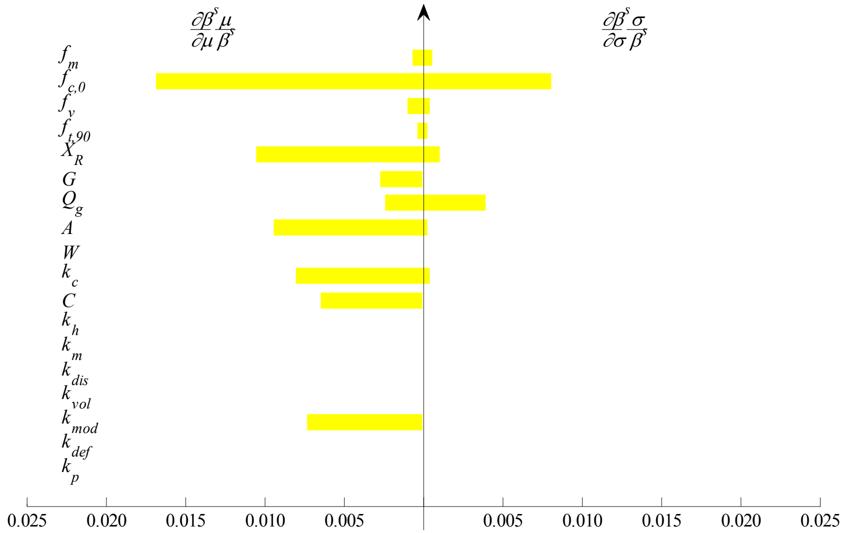
Reliability Analysis - Sensitivity Analysis

- System reliability index

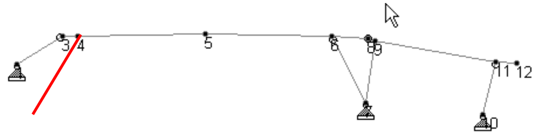
$$\beta^s \approx -\Phi^{-1}(P^s_f) \quad , \quad P^s_f = \left(\bigcup_{i=1}^{10} g_i \leq 0 \right) \approx 1 - \Phi_{10}(\bar{\beta}, \bar{\rho})$$

- Sensitivty with respect to statistical parameters

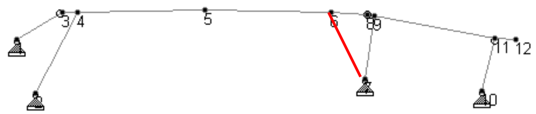


Removal of Key Elements

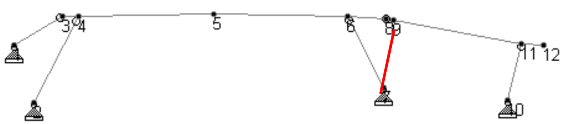
Failure of column 1-4



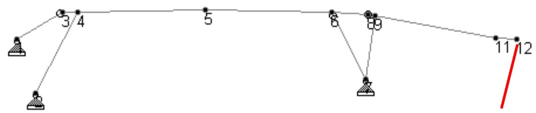
Failure of column 6-7



Failure of column 7-9

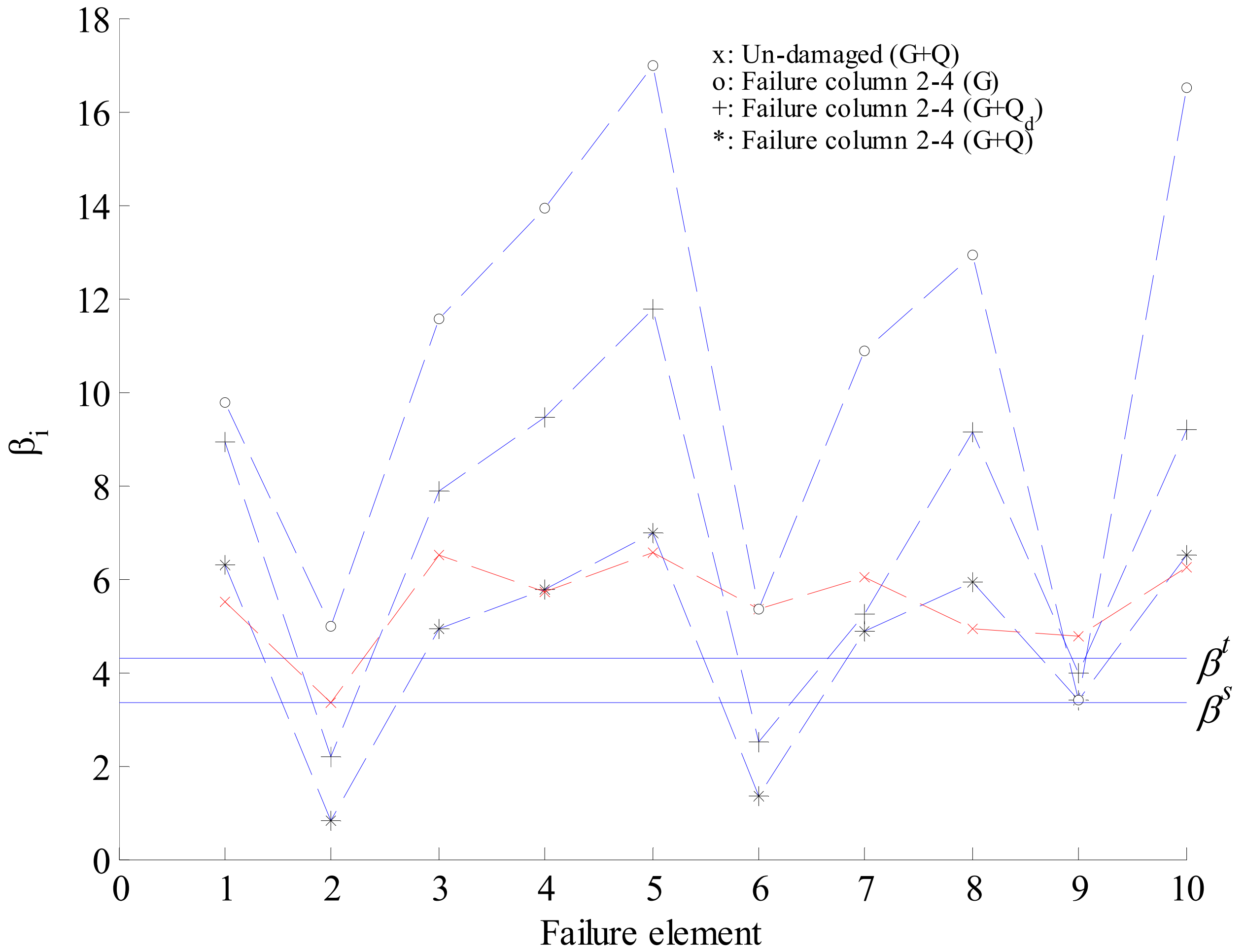


Failure of column 10-11

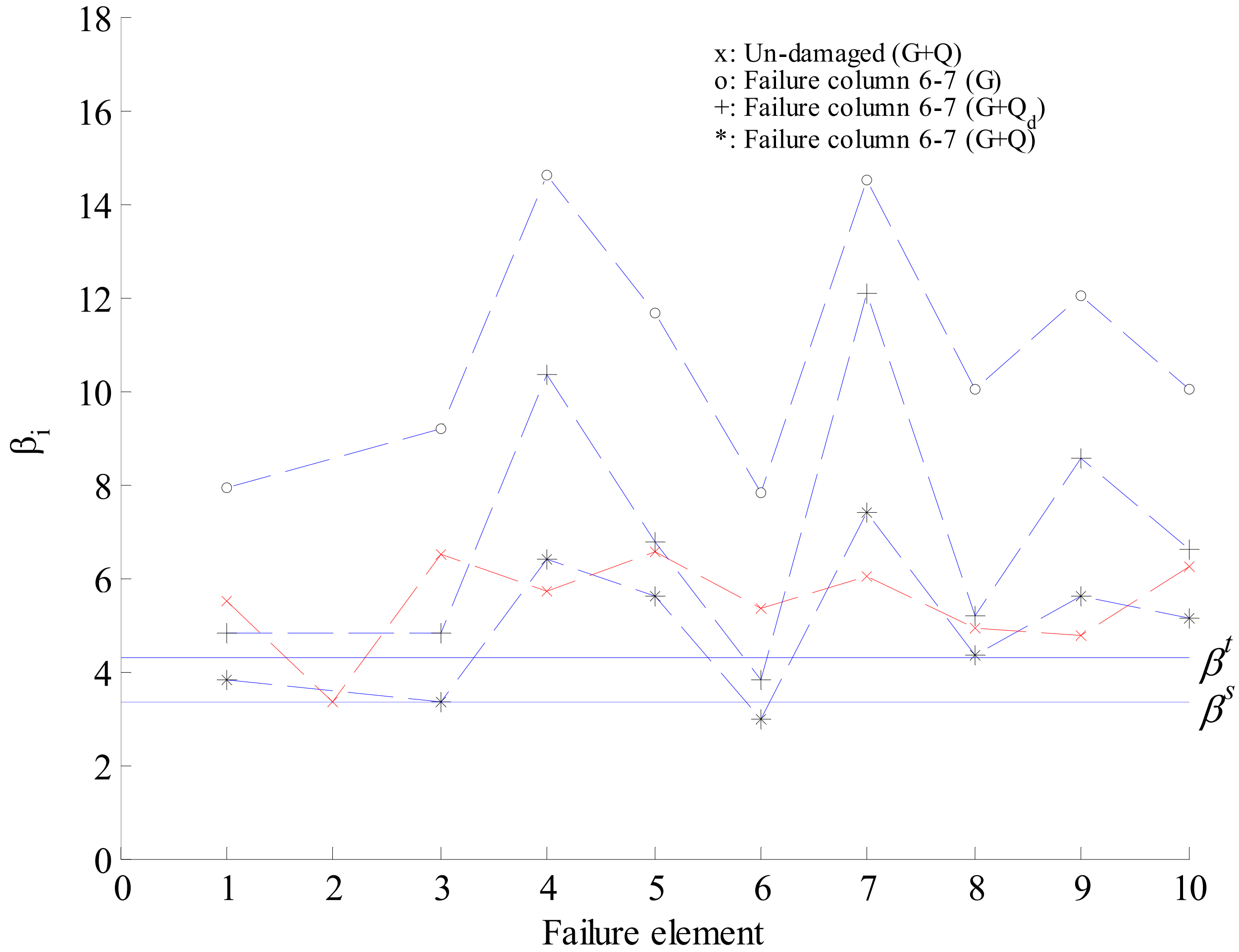


Each failure mode will be considered for the permanent load (G), permanent load and extreme snow (G+Q) and permanent load combined with a daily snow load (G+Qd) .

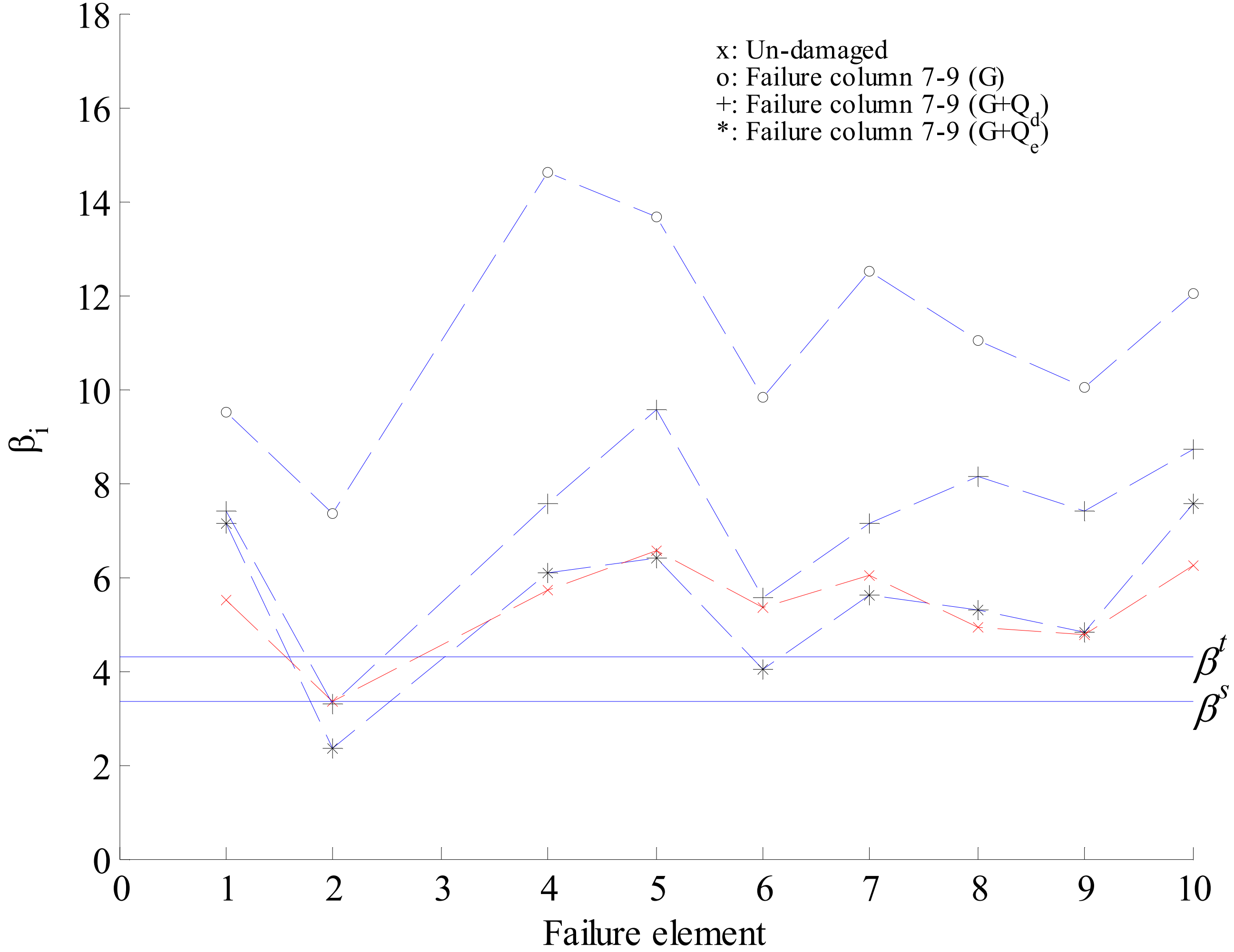
Removal of Key Elements – column 1-4



Removal of Key Elements – column 6-7



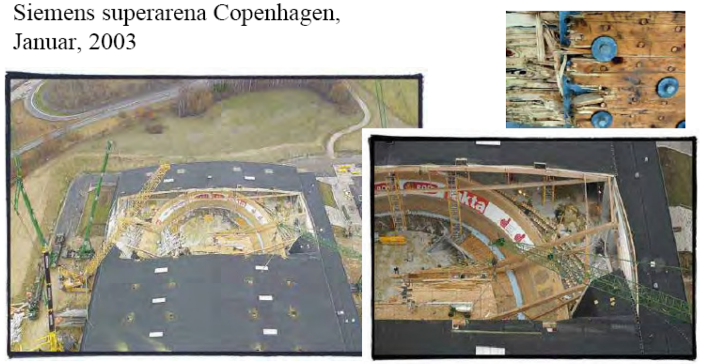
Removal of Key Elements – column 7-9



Conclusions

- The considered timber structure can be charaterised as robust with respect to the robustness framework used for the evaluation.
- However, related to use of the robustness framework, more analysis of requirements to target reliability, variable load and modelling is needed.
- Also a discussion of the definition extensive failure related to documentation of: *extensive failure of the structure will not occur if a limited part of the structure fails –*
- Acceptable collapse consequences

Siemens superarena Copenhagen, Januar, 2003



Future work

- Redundancy - Ductility (parallel system reliability analysis of frame), e.g. ductile failure of columns
- Non-linear material behaviour and in joints, sliding in joints
- Redistribution of load effects, system effects…
- Gross erros – unintentional load and defects

Thank You
for
Your Attention