COST E55 - Modelling the Performance of Timber Structures

3^{rd} Workshop and 5^{th} MC meeting

VTT Technical Research Centre of Finland

A 3D moisture-stress FEM analysis of timber structures

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Research projects

WoodFEM (VTT/Tekes)

Improved Moisture (WoodWisdom-Net 162006B)

Mechanical response of wood in presence of humidity

- Safety and serviceability of timber structures

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- Recent literature: ***3D computational wood mechanics*** (from end '90s to present. Implementation in Abaqus: Ormarsson *et al.*, Chassagne *et al.*, ...)

Orthotropic viscoelastic-mechanosorptive model

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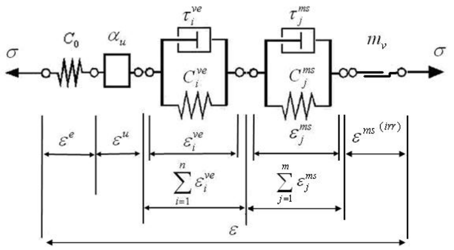

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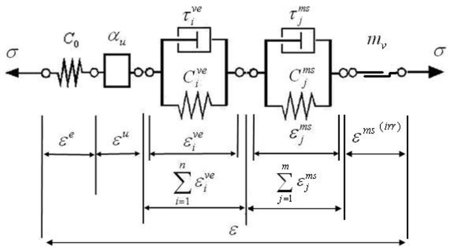

- **Wood as cylindrically orthotropic material**
- **Helmholtz free energy**

$$\psi(T, u, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_i^{ve}, \boldsymbol{\varepsilon}_j^{ms}) = \phi(T, u) + \frac{1}{2}\boldsymbol{\varepsilon}^e : \mathbf{C}_0 : \boldsymbol{\varepsilon}^e + \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{\varepsilon}_i^{ve} : \mathbf{C}_i^{ve} : \boldsymbol{\varepsilon}_i^{ve} + \frac{1}{2}\sum_{j=1}^{m} \boldsymbol{\varepsilon}_j^{ms} : \mathbf{C}_j^{ms} : \boldsymbol{\varepsilon}_j^{ms}$$

- **$\mathbf{C}_0$, $\mathbf{C}_i^{ve}$ and $\mathbf{C}_j^{ms}$: elastic, elemental viscoelastic and elemental mechanosorptive tensors. Kelvin elements.**
- **α_u: shrinkage coefficient; τ_i^{ve}, τ_j^{ms}, m_v: material parameters.**

3D creep matrices (first proposal)

- **Elemental matrices based on experimental results for wood parallel to grain (Toratti, 1992) and compliance parameters for cross grain wood sections (Hanhijärvi, 1997 and Toratti and Svensson, 2002)**

- **Differential equation for $\varepsilon_i{}^{ve}$**

$$\dot{\varepsilon}_i{}^{ve} + \frac{1}{\tau_i}\varepsilon_i{}^{ve} = \frac{1}{\tau_i}\mathbf{C}_i^{ve\,-1} : \boldsymbol{\sigma}$$

- **Viscoelastic compliances (Hanhijärvi, 1997)**

i	$C_{tang}^{ve}{}^{-1}$ [MPa^{-1}]	τ
1	0.00008	10^{-14}
2	0.00010	10^{-13}
..	...	..
11	0.00090	10^{-4}
12	0.00180	10^{-3}
..	...	..
20	0.15	10^{5}
21	0.25	10^{6}

- $C_{tang}^{ve}{}^{-1} \times \alpha;\ \alpha = 0.15$ for service conditions

- $C_{long}^{ve}{}^{-1} \times \beta$ with $\beta = 0.0128$

- **Irrecoverable mechanosorption**

$$\dot{\varepsilon}_j{}^{ms,irr} = m_v \boldsymbol{\sigma} |\dot{u}|; \quad m_v = 0.033\ [\mathrm{MPa}^{-1}]$$

- **Differential equation for recoverable $\varepsilon_j{}^{ms}$**

$$\dot{\varepsilon}_j{}^{ms} = \frac{\mathbf{C}_i^{ms\,-1} : \boldsymbol{\sigma} - \varepsilon_j{}^{ms}}{\tau_j}|\dot{u}|$$

- **Mechanosorptive compliances (Toratti and Svensson, 2002)**

j	$C_{tang}^{ms}{}^{-1}$ [MPa^{-1}]	τ
1	0.003	0.01
2	0.003	0.1
3	0.010	1.0

- $C_{tang}^{ve}{}^{-1}/10$ for this research tests

Update of the viscoelastic strain (Hanhijärvi and Mackenzie-Helnwein, 2003)

- Coupling $\Rightarrow$ serial decomposition scheme

$$\varepsilon^{ve}_{i,k+1} = \varepsilon^{ve}_{i,k} \exp\left(\frac{-\Delta t}{\tau_i}\right) + \mathrm{T_k}\left(\frac{\Delta t}{\tau_i}\right) \mathbf{C}^{ve\,-1}_i : \boldsymbol{\sigma}_k + \mathrm{T_{k+1}}\left(\frac{\Delta t}{\tau_i}\right) \mathbf{C}^{ve\,-1}_i : \boldsymbol{\sigma}_{k+1}$$

$$\Delta t = t_{k+1} - t_k$$

$$\mathrm{T_{k+1}}(\xi) = 1 - \frac{1}{\xi}[1 - \exp(-\xi)];\ \ \mathrm{T_k}(\xi) = 1 - \exp(-\xi) - \mathrm{T_{k+1}}(\xi)$$

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- Wood as **viscoelastic material** ⇒

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- Wood as **viscoelastic material** $\Rightarrow$
- Time-temperature-moisture superposition principle

$$\Delta t = e^a \Delta t;\quad a = k_T(T - T_{ref}) + k_u(u - u_{ref})$$

$T_{ref} = 100^o C$, $u_{ref} = 15\%$, $k_T = 0.095 \cdot \ln(10)\,[K^{-1}]$, $k_u = 43 \cdot \ln(10)$. (Master curves, Hanhijärvi, 1997)

UMAT user subroutine

- Variant of the incremental-iterative algorithm proposed by Mackenzie–Helnwein and Hanhijärvi (2003)
- **Stress increment at the current time step:**

$$\Delta\boldsymbol{\sigma}_{k+1} = \mathbf{C}_{\mathrm{T}}(\Delta\boldsymbol{\varepsilon}_{k+1} - \Delta\boldsymbol{\varepsilon}^{u}_{k+1} - \Delta\boldsymbol{\varepsilon}^{ms(irr)}_{k+1} + \sum_{i=1}^{k}\mathbf{R}^{ve}_{i} + \sum_{j=1}^{m}\mathbf{R}^{ms}_{j})$$

- **Tangent operator of the model:**

$$\mathbf{C}_{\mathrm{T}} = \left(\mathbf{C}_{0}^{-1} + \sum_{i=1}^{k}\mathbf{C}_{i}^{ve\,-1} + \sum_{j=1}^{m}\mathbf{C}_{j}^{ms\,-1}\right)^{-1}$$

- **Viscoelastic strain:**

$$\boldsymbol{\varepsilon}^{ve}_{i,k+1} = \boldsymbol{\varepsilon}^{ve}_{i,k} + \Delta\boldsymbol{\varepsilon}^{ve}_{i,k+1}; \quad \Delta\boldsymbol{\varepsilon}^{ve}_{i,k+1} = (\mathbf{C}^{ve}_{i})^{-1}\Delta\boldsymbol{\sigma}_{k+1} - \mathbf{R}^{ve}_{i}(\boldsymbol{\varepsilon}^{ve}_{i,k}, \boldsymbol{\sigma}_{k})$$

- **Mechanosorptive strain:**

$$\boldsymbol{\varepsilon}^{ms}_{j,k+1} = \boldsymbol{\varepsilon}^{ms}_{j,k} + \Delta\boldsymbol{\varepsilon}^{ms}_{j,k+1}; \quad \Delta\boldsymbol{\varepsilon}^{ms}_{j,k+1} = (\mathbf{C}^{ms}_{j})^{-1}\Delta\boldsymbol{\sigma}_{k+1} - \mathbf{R}^{ms}_{j}(\boldsymbol{\varepsilon}^{ms}_{j,k}, \boldsymbol{\sigma}_{k})$$

DFLUX user subroutine

- **Flow across the boundary** (Rosen, '78, Avradimis and Siau, '87):

$$q_n = \rho_0 S(u_{air} - u)$$

$$u_{air} = 0.01 \times \frac{-T\, ln(1-h)}{0.13(1-\frac{T}{647.1})^{-6.46}}^{\frac{1}{110T^{-0.13}}}$$

  - $S = 3.2 \times 10^{-8}\, e^{4u}$ m/s: surface emissivity
  - T: temperature in Kelvin
  - h: relative vapour pressure of air: $0.01 \cdot \%\mathrm{RH}$

Coupled moisture-stress analysis

Abaqus CAE

- geometry, material properties, material orientations, boundary conditions, mechanical loading, initial moisture fields, etc.
- 3D element types: C3D8T, C3D8R, C3D20T, C3D20RT

User subroutines

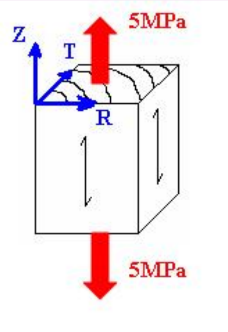
- UMAT, DFLUX and ORIENT (for curved beams)

Abaqus Standard

- moisture-stress analysis (analogy with the temperature-displacement analysis: validity of Fick's law)
- sequential temperature-moisture/stress analysis

Test 1 (Leivo, 1991)

- **Constant load, RH=35% = const, T=20^{o}C=const**
- ***Scott Pine* material parameters**



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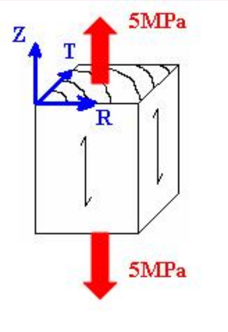
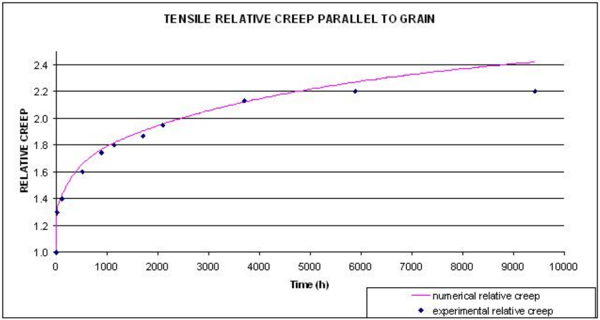
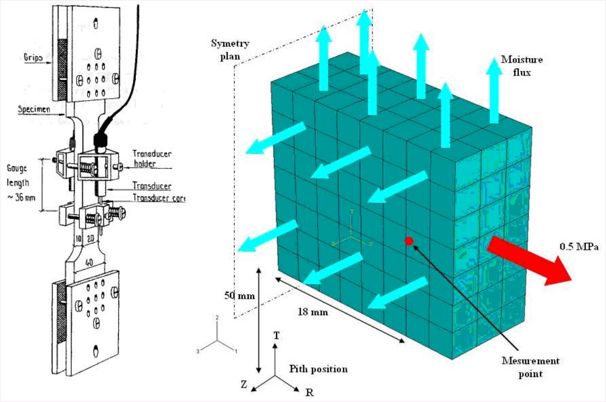


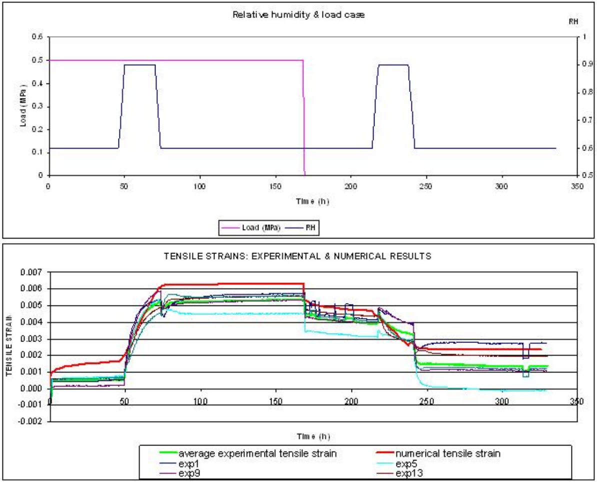


Test 2 (Svensson, 1997 - Short term analysis)

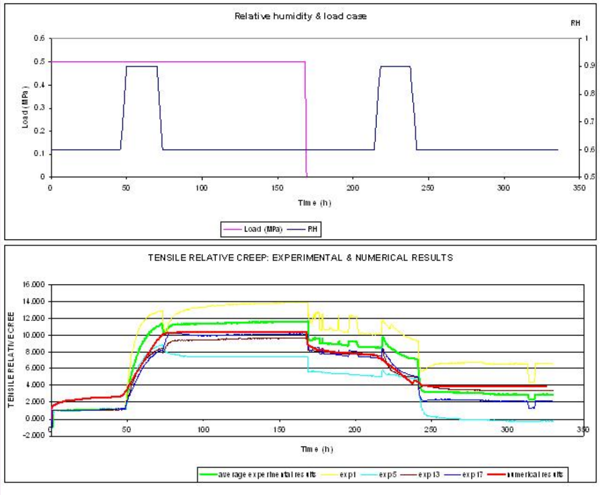
- **Load: 0.5 MPa=const for 168 h (tangential direction). Cycling relative humidity. T= 20°C=const. Load removed for the next 168 h**
- ***Scott pine* material parameters**



Results - Strain

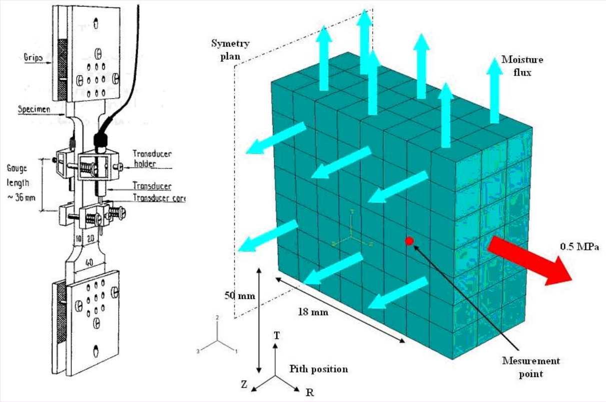


Results - Relative creep

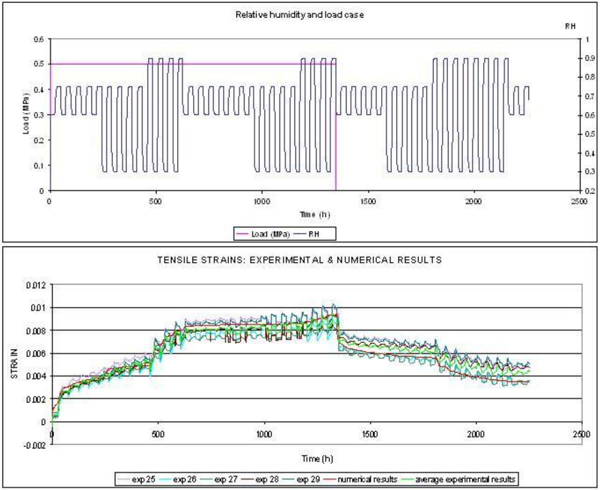


Test 3 (Svensson, 1997 - Long term analysis)

- **Load: 0.5 MPa=const for 1400 h (tangential direction). Cycling relative humidity. T= 20^{o}C=const. Load removed for the next 900 h**
- ***Scott pine* material parameters**



Results - Strain



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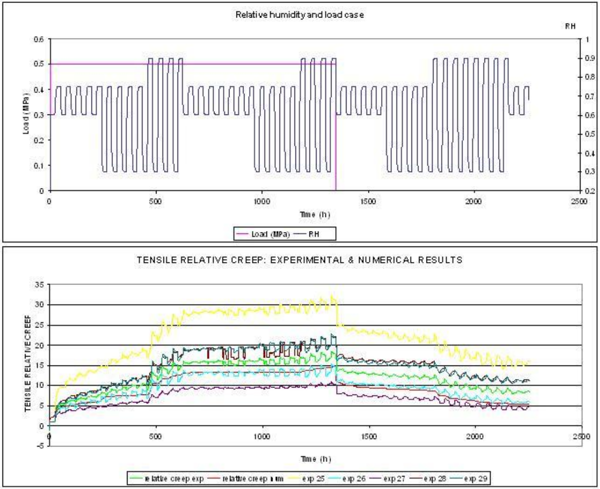
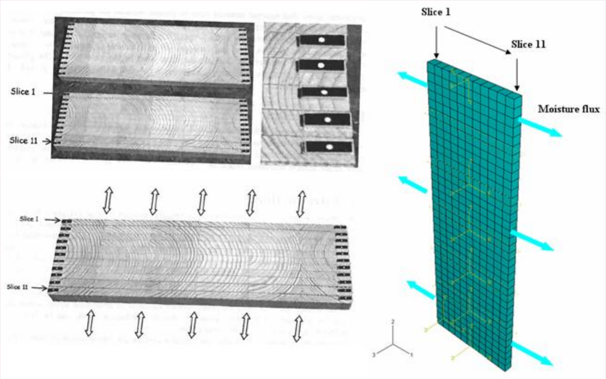


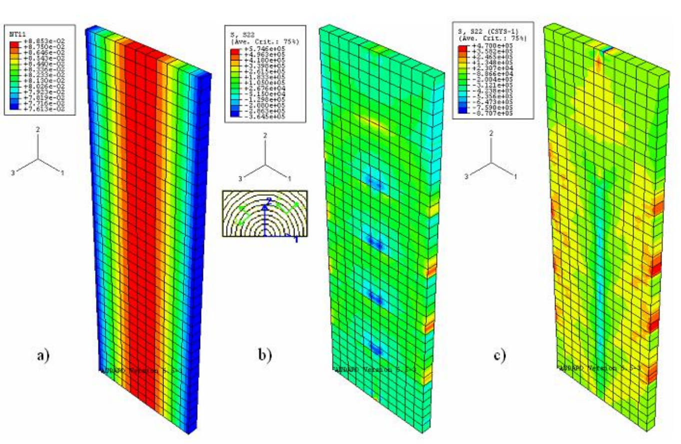
Test 3 (Jönsson, 2002)

- **Standard glulam beams (90X270, quality L40) containing 6 lamellas of *Norway spruce***
- **Glulam sawn into 16 mm tick plates; specimens free from knots** $16 \times 90 \times 270$ **mm**



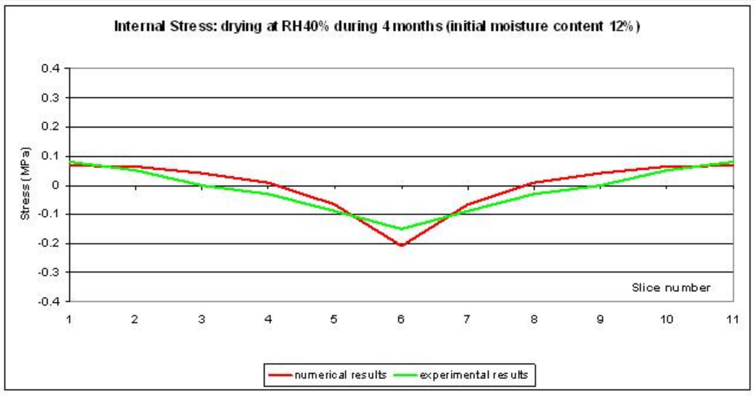
Results

Drying at RH=40%. Results after 4 months: a) moisture content, b) tangential stress (material coordinate system), c) vertical stress (global coordinate system)



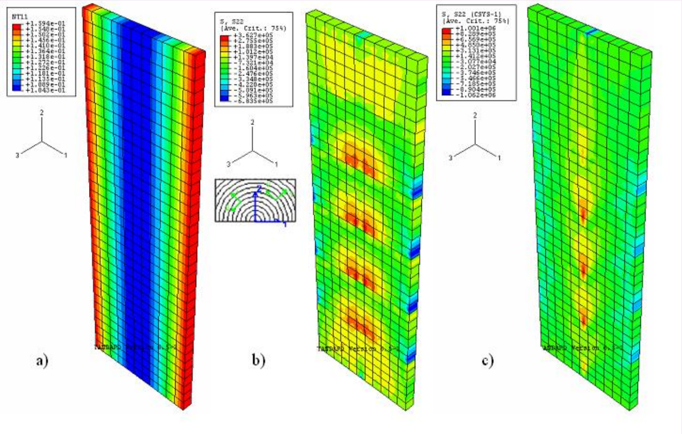
Results

Average of vertical stress for each slice



Results

Wetting at RH=80%. Initial moisture content: 4 months with RH=40%. Results after 38 days: a) moisture content, b) tangential stress (material coordinate system), c) vertical stress (global coordinate system)



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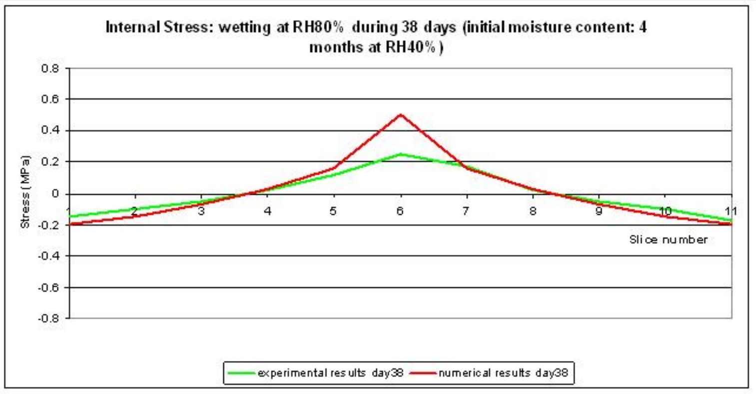

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- General method suitable for the evaluation of moisture induced stress in timber structures
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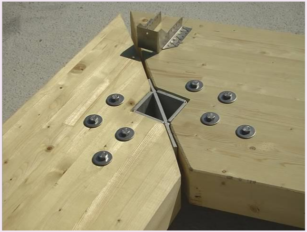

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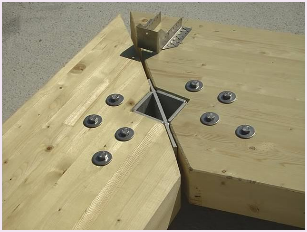

Future work

- Evaluation of moisture-induced stresses in timber joints
- Some work on modelling of failure in wood (onset of crack growth in presence of viscoelastic-mechanosorptive creep, etc.)