

Stress perpendicular to grain in glued laminated timber girders with special shapes

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Abstract

The paper presents the results of few researches in Croatia in the field of stress perpendicular to grain analysis inside glued laminated timber girders with special shapes. Researches include experimental part, analysis of parametrically prepared FE models, comparison of test results and theory, as well as results of design with various European codes and FEA. Also, as the result of this works the advises for the design and geometry limitation are given for the most common shape types of glued laminated beams.

1. INTRODUCTION

Glued laminated beams are often tapered and/or curved in order to meet architectural requirements, to provide pitched roofs, to obtain maximum interior clearance or to reduce wall height requirements at the supports. The most commonly used types are the single tapered beam, the curved beam with constant cross-section, the double tapered beam and pitched chambered beam (see Figure 1).

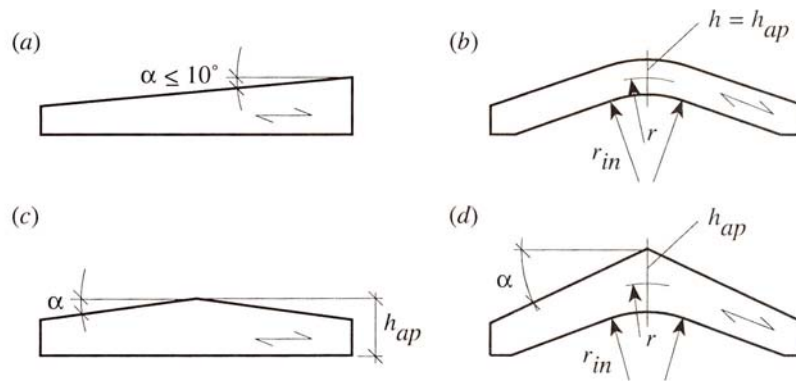


Figure 1 a) single tapered beam, b) double tapered beam, c) curved beam with constant cross-section d) Pitched chambered beam

As the result of their shape and the manufacturing process, these beams usually have parts with sawn taper cuts and apex zones with or without curved laminations. It is recommended that the laminations should be parallel to the tension edge of the beam with the tapered edges located to compression edge.

The distribution of bending stress in tapered beams is non-linear and therefore should be calculated using the theory of thin anisotropic plates, taking into account the ratios of E_0/E_{90} and E_{90}/G and Poisson's ratio.

Theory of the curved beam bending is based on the presumption that cross sections stay narrow after bending and perpendicular to the longitudinal axis of the beam after deformation. They have only relative rotation. Laminated girders with small curvature (or non-curved) longitudinal axis, has linear distribution of the stresses, while gravity centre is in the neutral axis. Members with large curvature (ratio $0,1 \leq h_{ap} / R < 0,5$, while $r_{in} / t \leq 240$, etc. lamelas are not optimally curved and consequence of this is that such girders behave different in bending Changes can be easily saw on differntial part of curved member defined with angle dp (Figure 2). For the purpose of the design, maximum bending stress at the tapered edge can be calculated approximately according to simple bending theory modified by factor $k_{m,\alpha}$, depending on the slope of the face and the fact if the tensile or compressive stresses are on the tapered edge.

In the apex zone of curved and tapered beam the distribution of the bending stress is also non-linear and the distribution of the bending stresses is also nonlinear. Additionally, radial stresses perpendicular to grain are caused by bending moments. On the Figure 2 is incremental section of a curved beam to illustrate the distribution of the bending stresses. The fibres on the inner side of a beam are shorter than those on the outer side, the distribution of bending stress is therefore nonlinear and hyperbolic with the maximum stress at the inner fibre. Based on Navier's theory and assuming the neutral axis at mid-depth the strains are as follows:

$$\varepsilon_i = \frac{\Delta dl_i}{dl_i} > \varepsilon_e = \frac{\Delta dl_e}{dl_e}$$

According to that $|\sigma_i| > |\sigma_e|$ equilibrium of the internal forces over the cross-section is only possible if the neutral axis is closer to the inner edge.

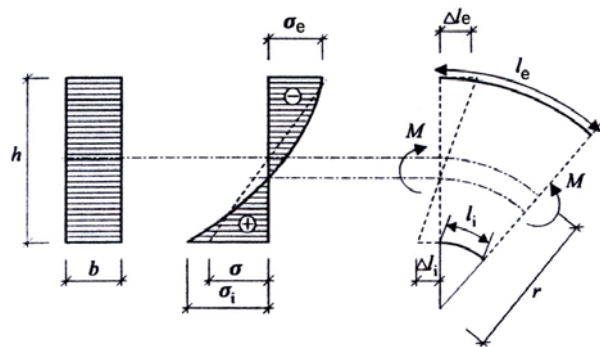


Figure 2 Distribution of bending stresses in a curved beam

Bending moments in curved members cause radial stresses perpendicular to grain (Figure 3). Assuming, for simplification, a linear stress distribution, it can be easily shown that resulting tensile and compressive forces F_t i F_c , lead to the force U in radial direction. If the moment increases the radius of curvature, the radial stresses are in tension.

$$F_c = F_t = \frac{1}{2} \cdot \frac{h_{ap}}{2} \cdot \sigma_m \cdot b = \frac{b \cdot h_{ap}}{4} \cdot \sigma_m$$

$$U = F_c \cdot d\varphi = F_t \cdot d\varphi$$

Tensile force U is taken by area $A = b \cdot ds = b \cdot (R \cdot d\varphi)$ so the tensile stress perpendicular to grain are:

$$\sigma_{t,90,max} = \frac{U}{A} = \frac{U}{b \cdot (R \cdot d\varphi)} = \frac{h_{ap} \cdot \sigma_m}{4 \cdot R \cdot b} = 0,25 \cdot \frac{h_{ap}}{R} \cdot \sigma_m = k_p \cdot \frac{M_{ap}}{W_{ap}}$$

Shape factor k_p is used to modify tension stresses perpendicular to grain according to the geometry.

$$k_p = 0,25 \cdot \frac{h_{ap}}{R}$$

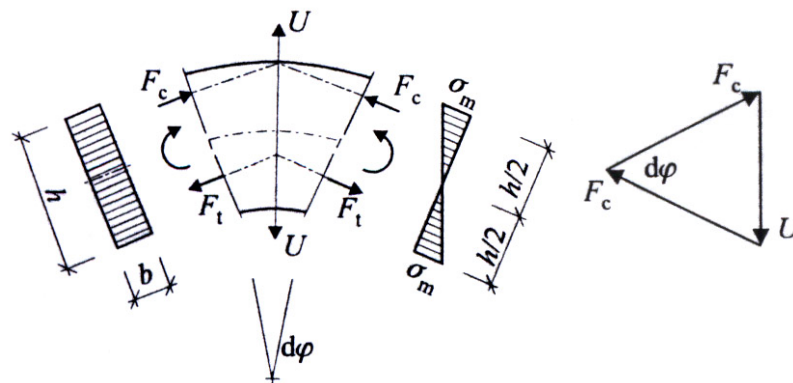


Figure 3 Theoretical explanation of tensile stress perpendicular to grain in curved beam

2. ANALYSIS OF FIVE TYPES OF GLULAM GIRDERS WITH AND WITHOUT FRP REINFORCING PLATES

Inside the scientific project “Analysis of reliability of the timber structures” led by Prof. Žagar, candidate M. Haiman (both from the Faculty of Civil Engineering, University of Zagreb) made the dissertation [5] about reliability of glulam beams with or without reinforcement with FRP plates. Following (Figure 4) present five types of glulam which were taken into analysis with and without FRP plates glued on side in direction perpendicular to grain. Research has done by lab tests which were carried out inline with detailed FE numerical analysis of whole glulam beams. Beams were not in everyday practice size but scaled to smaller dimensions as it could be seen on Figure 4. The aim of the research was to spread the knowledge about possibility of raising the safety and the bearing capacity of glulam beams using CFK (FRP) plates reinforcement method in regard to the complex state of stresses. Another aim was to determine “rolling-shear” bearing capacity of grain. Numerous FE test models were done for unreinforced and for the CFK plates reinforced glulam beams and were compared to real lab test model behaviours. There were used 2D and 3D FE models. Generally there were two kinds of specimens: first with straight-line intrados and second with radial curved intrados.

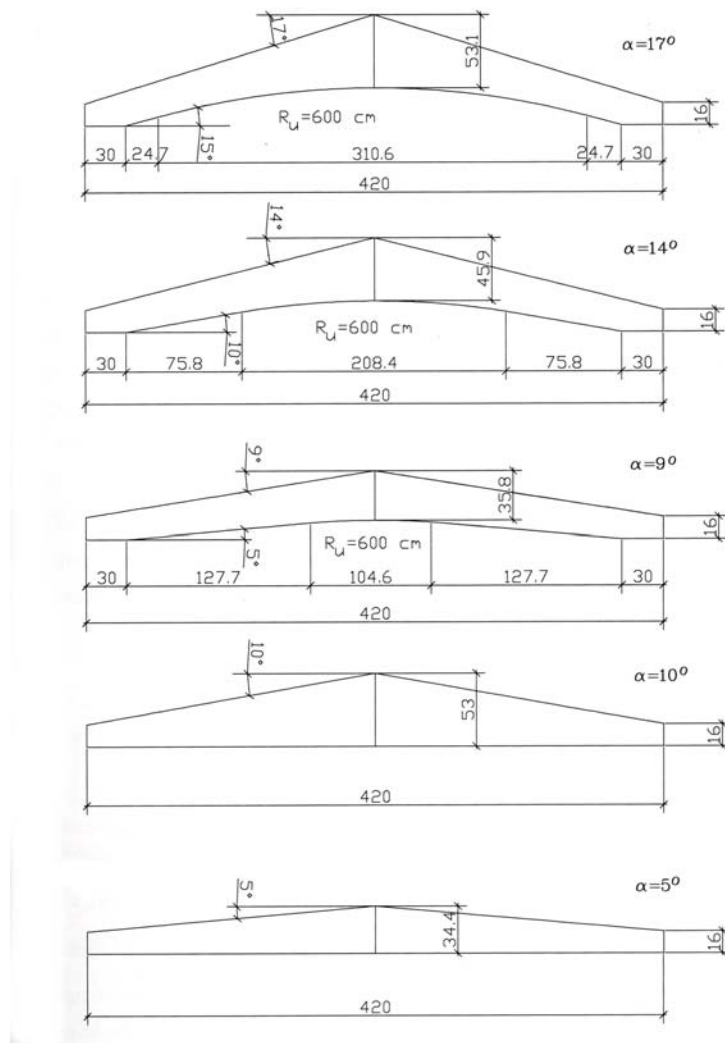


Figure 4 Types of analysed glulam beams with special shapes

2.1. Analysis of the “rolling shear” of the timber grains in contact with FRP plate

For that purpose, five samples for every type of glulam girders were tested. Ultimate and design value of rolling shear strength were determined. Next Figures show the test setup, mode of sample collapse and theoretical deformation of the tracheae of the timber under tensile force on the FRP plate.



Figure 5 Test of the samples – moment of collapse

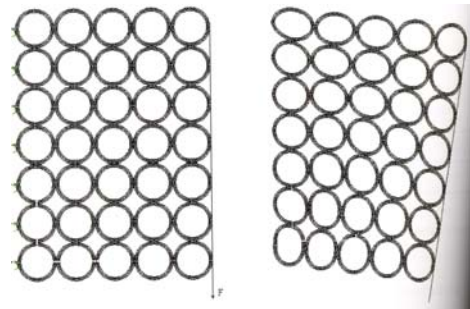


Figure 6 Tested samples – collapse across wooden surface(up); Model of tracheae in wood under tensile force from FRP plate(down)

From the Figure 4 and 5 it could be seen that the collapsed mode is not in glued line and that the “rolling shear” manifested as tensile stress perpendicular to grain which caused failure in wood.

2.2. Lab tests of the glulam girders

As it could be seen, five types of glulam beams with special shapes are tested on the bending according to four point test (Figure 6). Measuring points are shown on the Figure 7. With number 2 spots for deformations measurement are marked and with number 3 four spots with inductive tens meters.

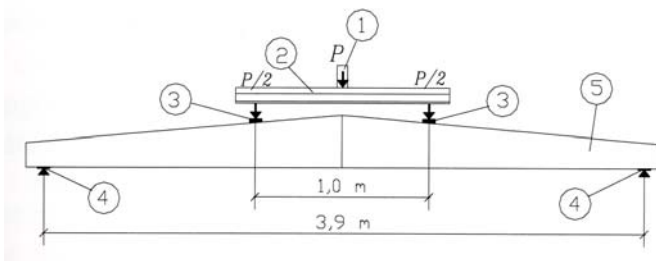


Figure 7 Set up for four point bending test

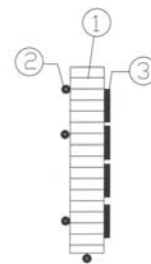
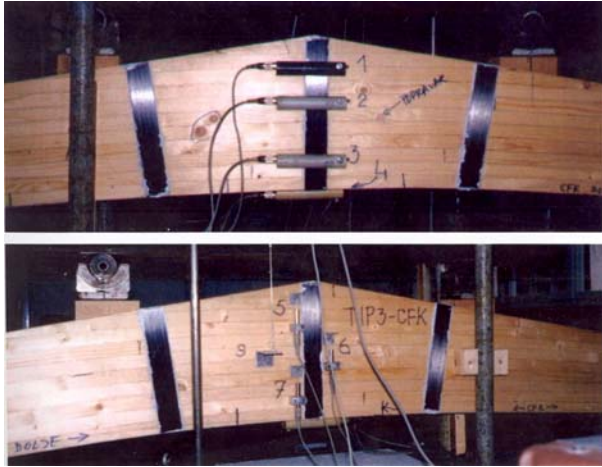


Figure 8 Measuring spots showed on cross-section

Figures 9 & 10



Position of the FRP plates: plates were in the apex of the glulam girders but also they were glued 40 cm from the midspan on both sides (left); Collapse of the glulam beam without FRP plate due to stress perpendicular to grain (right)



2.3. Numerical analysis of the tested glulam girders

All dimensions of the numerical analysed girders followed the geometry of lab tested girders. For numerical modeling finite element SHELL4L was used because it has possibility to define orthotropic material as wood is. To increase the accuracy of analysis, QUAD4 option was chosen and list of results in the local coordinate system. It is especially important because the results of stresses σ_x and σ_y are shown parallel to down edge and perpendicular to down edge.

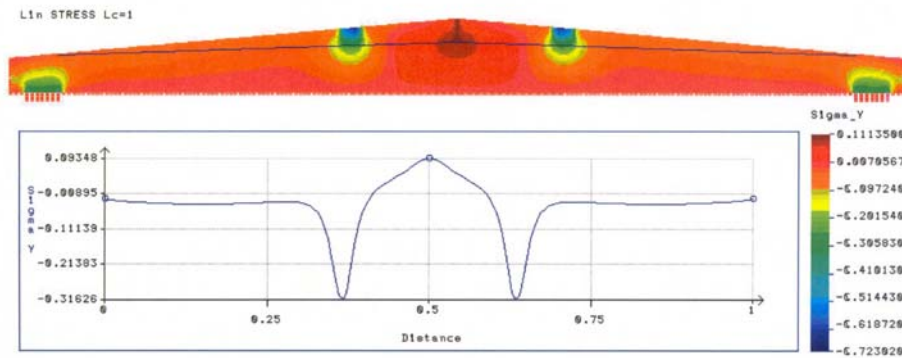


Figure 11 Distribution of the stresses perpendicular to grain in tapered beam, longitudinally - TYPE 1

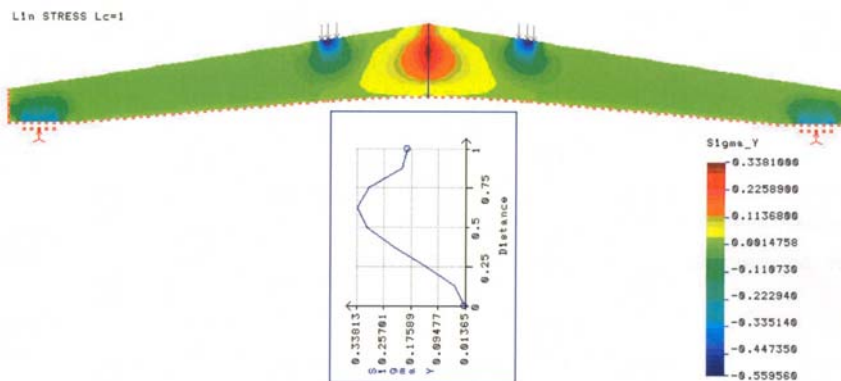


Figure 12 Distribution of the stresses perpendicular to grain – in apex cross section – TYPE 3

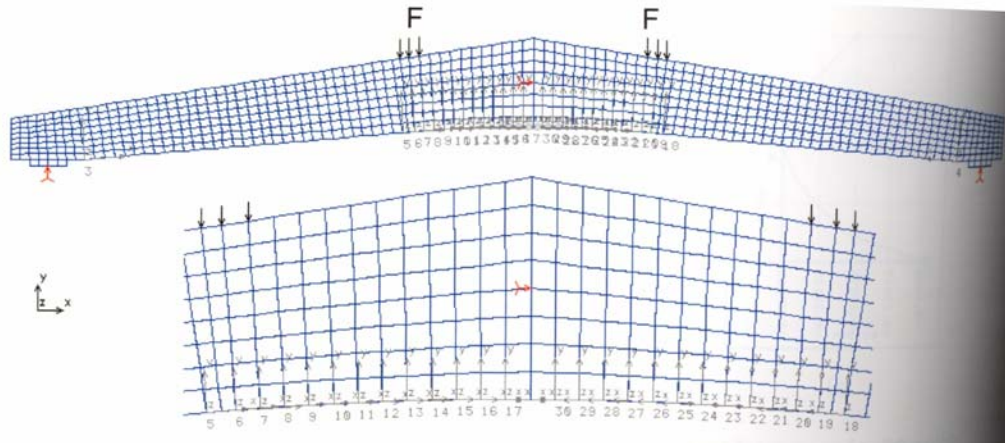


Figure 13 Finite element mesh of the glulam girder TYPE 3, with magnified part of many local coordinate system on down edge of the girder

It is important to emphasise that orthotropy essentially change the state of stresses perpendicular to grain as well as distribution of the shear stresses. So, in numerical analysis it is very important to use the program packages which permit analysis of structures from orthotropic materials. Figure 12 shows detailed modeling of the material characteristics parallel to down edge using numerous local coordinate systems. The difference between correct model and incorrect, very rough model could be very big looking in percent (up to the 100%). When the model is correct, tensile stresses perpendicular to grain decrease and it is very near to the value of the measured stress.

2.3. Conclusions

As one of the results of the lab test, the following tables show description of the failure mode for every type of glulam laminated girder with or without reinforcement with FRP plates.

Description of failure mechanism on glulam beams without reinforcing FRP plates	
TYPE 1	Defect in wood is spotted at finger joint in outer lamela in tensile zone. Failure started from this spot. Complex state of stresses caused developing of crack and failure.
TYPE 2	From the defect in wood in tensile zone, failure crack started. The failure mode suggest that failure is caused by exceeding shear strength from the spot of the detected defect.
TYPE 3	Failure due to exceeded tensile stresses perpendicular to grain.
TYPE 4	Failure crack started from the knot in tensile zone across tensile arch zone of the whole beam. The reason for failure is complex state of stress.
TYPE 5	The same situation as with TYPE4
Description of failure mechanism on glulam beams reinforced with FRP plates	
TYPE 1	The failure started at knot in the zone of concentrated stresses at the upper end of the FRP plate. Failure crack developed due to complex state of stresses.
TYPE 2	Failure started at finger joint near the end of FRP plate. Failure crack developed due to complex state of stresses.
TYPE 3	Knot near the down end of the FRP plate initiated concentration of the stresses and failure. Failure crack is simple and tensile stresses perpendicular to grain are not responsible for failure as in case of type 3 without reinforcement
TYPE 4	Both sided longitudinal FRP plates in tensile zone form stiffer body. Failure started in knot and the failure crack is from that spot, perpendicular to timber grains
TYPE 5	The same situation as with TYPE4

After the conducted research on real non-reinforced and reinforced glulam beams with special shapes with FRP plates the results are:

1. In almost every case, failure of the beam started because the defect point in wood (natural defect like knot or bad finger joint)
2. Measurements of the deformations ε_x and ε_y in all five types of glulam beams shows good correspondence with the results of numerical analysis done by COSMOS/M program package. This gives the opportunity for reliable real modeling of glulam girders using such packages.
3. Good correspondence of numerical and testing results of the deformation ε_x was in range $\pm 10\%$. For the perpendicular deformations ε_y range was bigger ($\pm 20\%$).
4. According to P- δ diagrams it could be seen that the glulam girders with bigger volume and all girders with FRP plates reinforcement are ductile after force $2/3 F_u$.
5. In four types of glulam beams, putting the FRP plates in lower tensile zone longitudinally gives much more reliable beam. On the other hand, glueing the FRP plates sided to glulam girder is much less effective than it was expected before testing.
6. In the glulam beams with width of 10, 12, most 14 cm, tensile stress perpendicular to grain could be partly effectively taken over by FRP plates.
7. Lab tests has shown that the tested wood is in the range from class BS11k to BS14k. Modulus of elasticity corresponde with class BS11k.
8. Very usefull research of the perpendicular „rolling shear“ capacity showed almost identical strength as $f_{t,90,k}$. By the space model analysis it is shown that depth influence of the FRP plates is about 20 mm which is not enough for the glulam beams of mid and larger span.
9. Putting the FRP plates longitudinally in the bottom of tensile zone across the whole span is very good solution which increase the level of reliability significantly, especially when it covers defects in wood.

3. ANALYSIS OF SLOPE EFFECT AND GEOMETRY OF GLULAM PITCHED CAMBERED BEAMS – ADVISES FOR THEIR DESIGN AND PRACTICAL APPLICATION

This part of paper represents a part of diploma work of student Vedran Pavlic under the mentorship of A. Bjelanovic, PhD, in the scope of glued laminated girders of special geometry. It also shows results of previous research engaged in the field of timber (material and structural design) and FEM. The main subjects of interest are glulam pitched-cambered beams the form of whom is aesthetically very impressive. However, an unsuitable form, e.g. inappropriate slope, due to the span and radius of curvature of the apex zone, could incite very serious problems in practice and jeopardize a safety of the whole structure. Wood is an anisotropic material whose tensile strength perpendicular on grains is far below the rest of its material properties. Therefore, it has to be deal very seriously with the safety of the curvature zone of girder, where these stresses appear. EC5 makes some simplifications that are acceptable enough in engineering practice. the results obtained using different codes and FEA are compared.

3.1. Introduction

A big intrados curvature in the ridge zone, non linear distribution of stresses along the cross section's width, and both a complex state of stresses caused by girders' geometry and anisotropy of wood as a material make it necessary to handle carefully the calculation of those girders. As for their mechanical resistance and stability, the study of tensile strength perpendicular to grains intensified by curvature is of enormous significance. The bearing capacity of wood in regard to tension perpendicular to grains is exceptionally low and varies significantly from its other mechanical properties. It is therefore hard to optimize the dimensions of those girders according to this criterion as its other characteristic cross-sections are not enough used. Such a problem can be avoided in different ways: local strengthening of the curved area, the construction of ridge girders of reduced static height (rounded ridge) or the construction of girders in couples instead of uneconomic increase of the single girder's height. Much better solutions relate to the geometry's rationalization (the slope of the flat girder's part of the axis, the raise of the curved area radius and the variation of the static height – full or reduced, modifying or constant girder's height as well) in relation to the span. The paper advocates such an approach, and the results lead to the summary of guidelines for a practical usage. As the basis for an investigation new European norms were taken, EN 1995:2004-1-1. We compared the results with those of the calculation according to pre-norms, DIN ENV 1995:2000-1-1. The proofs for mechanical strength and stability of the characteristic section x-x relevant for the stability control differ in

the norm and pre-norm. The FE 2D and 3D girders' models are parametrically prepared as well, undergoing the static analysis and global stability analysis (buckling) in order to look at the impact of simplification which leads to the introduction of norms into the calculation. The expressions for dimensioning (norms) by a series of coefficients transform the non linear stress distribution with respect to the height of the cross-section into a linear one, and a complex stress state transforms into bending stresses. FE analysis done in COSMOS/M package reflects a real state of stresses and deformations. Therefore, we wanted to investigate whether, and to which extent are the practical proofs of mechanical strength and stability on the side of safety. The paper addresses just a small part of research deals with investigations in the field of limits of suitable geometry on certain spans.

3.2. Description of methods included in research

The constants in the calculation are as follows:

1. Mechanical properties (strength, stiffness and density modules) of comparable classes of laminated wood with high bearing capacity and of 1st serviceability class – BS 28H (DIN ENV 1052:2000) made from structural timber S13, and GL 28H (EN 1995:2004) made from structural timber C30 class.
2. The 5,0m distance of main girders and characteristic load of 1,25kN/m² as a short - term action. The total value of the constant action has been estimated to be 0,7kN/m².
3. Symmetric load of main girders (the wind impact omitted – asymmetric impact).

The application of programme packages and the approach to the calculation (parametrically conducted analyses):

1. Programme package MATCHAD 13 for the proofs of mechanical strength and girders' stability. The girders are modelled parametrically (seven input parameters are: intrados and extrados slope angles, α and β , their divergence, γ , L girder span, chord length on the C_{in} intrados dependent on the radius and central curvature angle, affecting the volume of the curved zone, then cross-section dimensions – the height on the girder's bearing, h_a and the width, b). The alteration of stresses and the estimation of the cross - section 's bearing capacity (satisfying or not) result from varying the input parameters.
2. The Excel programme package has been used for limiting the input parameters values ($\alpha, \beta, \gamma, L, C_{in}, h_a, b$) so as to evaluate the girder's rationality. The assessment of rationality (dependence of mechanical strength and girder's stability on geometry). With regard to limitations of the study scope (samples' number, simplified analysis only for symmetric actions, etc.), rationality assessment for the glulam girders of highly curved intrados can be considered as a design «guideline».
3. The COSMOS/M programme package has been used for MKE analysis of parametrically modelled girders (the same input parameters) undergoing the same analysis and repetitive calculations. The package is used to introduce the possibility of comparing results with those obtained by the calculations and application of expressions for dimensioning (EN, ENV).

3.3. Parametric design procedure obtained in accordance with EC5 codes

The calculation's procedure diagram shows the ways in which the girder's calculation and the analysis of design's rationality along with the construction of related girders were conducted. The work presents the required stages to dimension accurately and rationally the girder (pitched camber and/or curved girder of high intrados curvature). The girders are simple static schemes (freely leaning beams), and the curvature is in the limits of $2 \leq R / h_{ap} < 10$, tj. $0,1 \leq h_{ap} / R < 0,5$. The design of these girders is technologically far more complex as the axis in the ridge zone is designed by inserting the short radius curve. The number of phases and the mode of gluing the plates affect the static height and the bearing capacity in the ridge zone (full or reduced cross-section of pitched cambered girders). However, the height in the ridge exceeds 3,0m of the production hindrances (the limit value is here restricted to 2,5m). The curved area's volume limitation is 2/3 from the total girder volume. The biggest transversal slope of these girders limits to the value $\leq 25^\circ$. There is also an additional limitation related to the value difference $\alpha - \beta \leq 10^\circ$ for the pitched cambered girders, the cross-section height of whom changes due to varied slope angles of extrados (α and intrados (β). Such a condition results from the fact that the plates are, generally laid parallel with one of the girder's generatrices. Therefore, the effect of oblique slashed edge appears on the opposite generatrix.

The unsuitable effect that oblique weakening has on the values of components of complex stress state gets modified by limiting the generatrices' slope difference.

In the practice, the shapes of laminated pitched cambered girders of curved intrados can be constructed so that their static cross-section height in the ridge zone gets reduced (fitted apex), whereas the constant remains (concentric curvature). In the zone where the girder's axes the straight line's height of girder's cross-section can be either changeable or constant. Due to the reduction of static height in the ridge zone they are treated as curved girders.

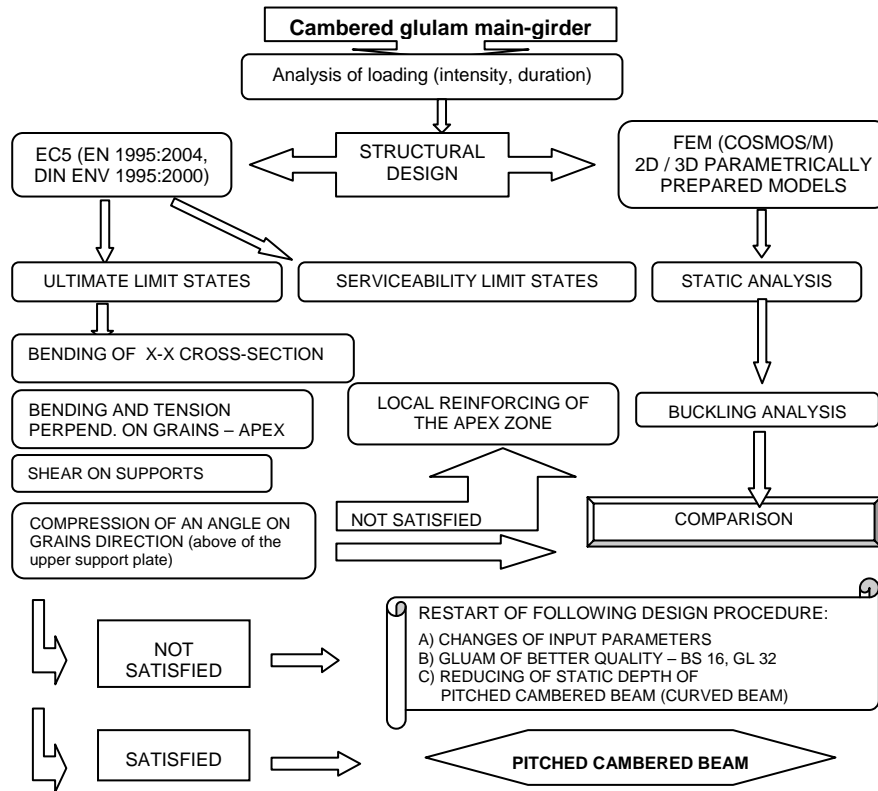


Figure 14 Flow-chart of design procedure and parametric analysis

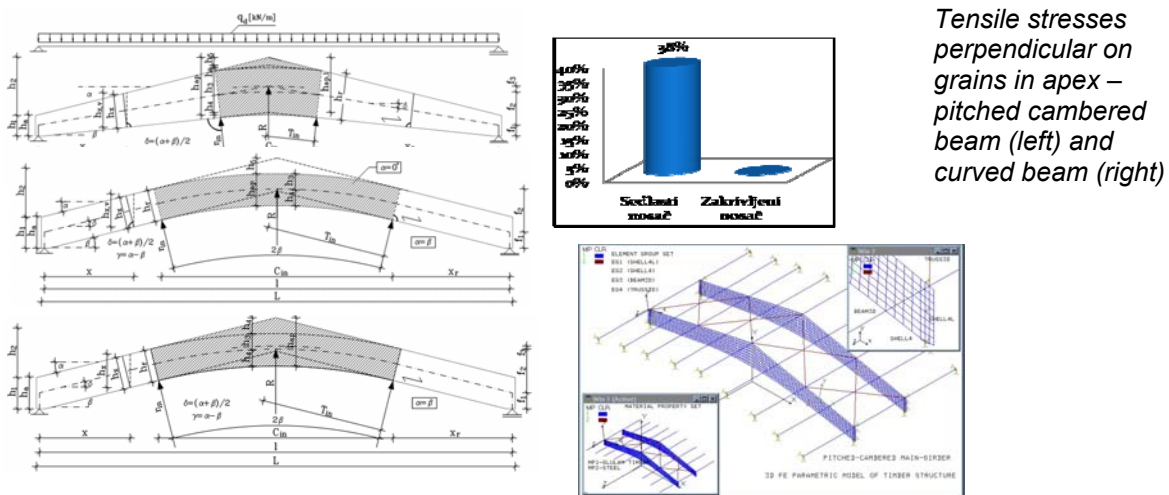


Figure 15 pitched cambered beams and curved beams (cambered beams with non-glued apex – Reduced static depth of cross-section) on left. 3D parametrically prepared FE models – SHHELL4L (girder), BEAM 3D (purline), TRUSS 3D (bracing diagonals) and PLATE FE (just for upper support plates) on right.

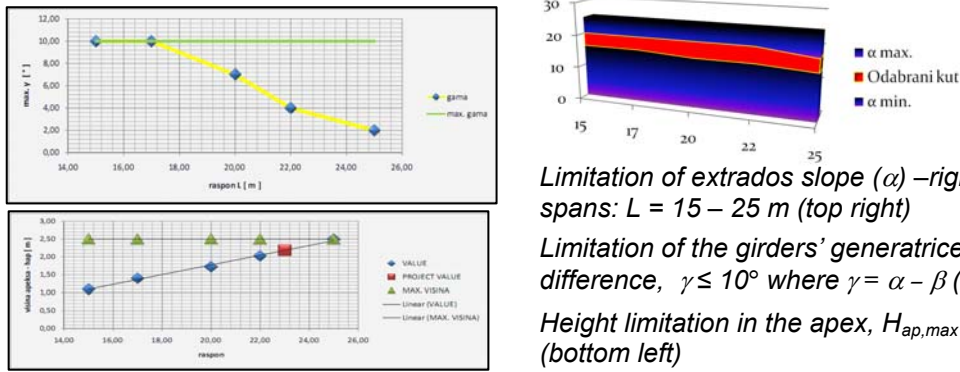


Figure 16 Input parameters, values of limitations and obtained results

The comparison of the design according to ENV and EN does not show major differences in the utilisation and safety. The difference in the value of partial coefficients for material ($\gamma_M = 1,25$ according to EN or $\gamma_M = 1,3$ according to DIN ENV is negligible). As far as the calculations are concerned, the only difference relates to the treatment of the characteristic cross-section x-x on the girder's flat part, where the difference in the cross-section utilisation (bending and stability), in the notched extrados is 30% – 36%, and the calculation according to EN 1995-1-1:2004 (eq. 1, 2) is on the safety side in relation to DIN ENV 1995-1-1:2000 (eq. 3,4). The difference in the utilization of cross-section x-x on the lower flat intrados is little and amounts some 15%. The comparison with FEA shows that the impact of longitudinal force (visible in FE models) for the girders with a usual bearing combination (unmovable and movable) is negligible. Therefore, the simplifications applied in the norms are practice related and acceptable from the engineering point of view, in laboratory research and the theory of anisotropic plates. The cross-section positions x-x with the highest stress from bending, comparing FEA and EC5 are not the same.

$$\sigma_{m,d,\gamma} = \frac{M_{x-x,d}}{W_{x-x}} \leq k_{m,\gamma} \cdot f_{m,d} \quad (1) \quad k_{m,\gamma} = \frac{1}{\sqrt{1 + \left(\frac{f_{m,d}}{1,5 f_{v,d}} \cdot \text{tg} \gamma\right)^2 + \left(\frac{f_{m,d}}{f_{c,90,d}} \cdot \text{tg}^2 \gamma\right)^2}} \quad (2)$$

$$\sigma_{m,d,\gamma} = (1 - 4 \text{tg}^2 \gamma) \cdot \frac{M_{x-x,d}}{W_{x-x}} \leq k_{m,\gamma} \cdot f_{m,d} \quad (3) \quad k_{m,\gamma} = \frac{1}{\frac{f_{m,d}}{f_{c,90,d}} \cdot \sin^2 \gamma + \cos^2 \gamma} \quad (4)$$

3.4. Conclusion

For the above described cambered beams the vital bearing capacity proof for tension perpendicular to grains in the apex, especially when there are greater slopes. By connecting the stress lengths perpendicular to grains obtained from EC5 and FEM we get to the difference in the distribution and values of the results, which is confirmed by the connection with the slope. Both design methods include conforming to the stress increase trend, but not with the stress magnitude. The design of pitched cambered girders is not rational on the $L > 25\text{m}$ spans. The better solutions present the curved girders ($L < 30\text{m}$) or the full replacement of static system.

4. PARAMETRIC ANALYSIS OF SLOPE EFFECT AND GEOMETRY OF GLULAM PITCHED CAMBERED BEAMS

4.1. Parametric design procedure in Excel program

Using Excel program, parametric analysis of glulam pitched cambered beams is conducted. This kind of girders have usual span between 10 and 30 meters, so this span range is analyzed. Load is constant (13 kN/m) for each analyzed span (to fully asses only importance in geometry change). Timber class is also same (BS 14) for each span. Thickness of girder is 20 cm and lamina thickness is 30 mm. Figure 16 represents all the variables that were used during parametric modeling.

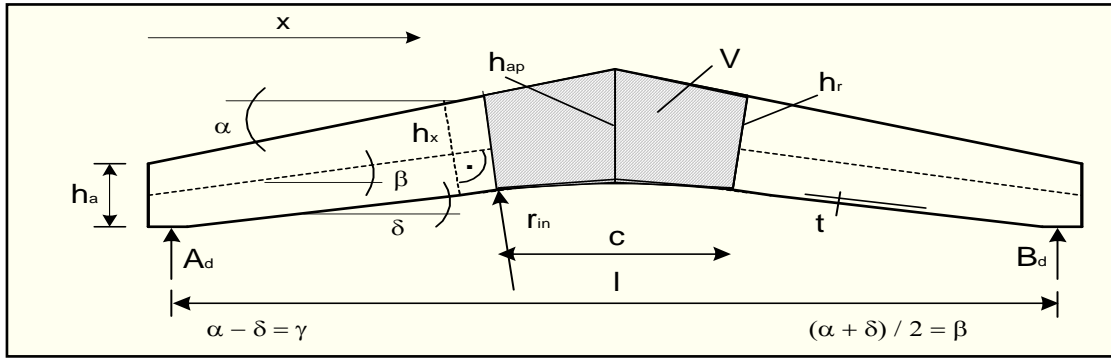


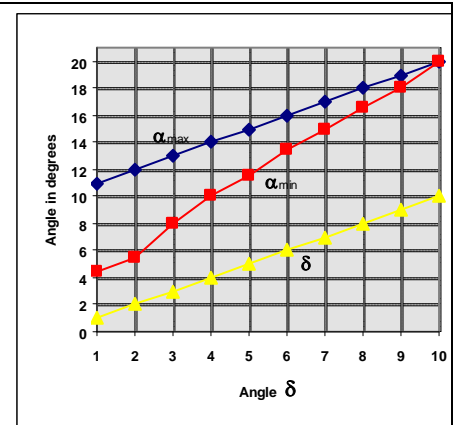
Figure 17 Variables for parametric modelling

Three different spans are analyzed: 10, 15, 20, 25 and 30 meters.

a) Span: 10 meters

Table 1. Values of different geometry variation parameters and their dependence concerning δ angle

α_{max}	α_{min}	δ	$\gamma_{min} = \alpha_{min} - \delta$	$\gamma_{max} = \alpha_{max} - \delta$	r_{in}	h_{ap}
11°	4,5°	1°	3,5°	10°	86,00 m	0,62 m
12°	5,5°	2°	3,5°	10°	43,00 m	0,63 m
13°	8°	3°	5°	10°	28,70 m	0,78 m
14°	10°	4°	6°	10°	21,50 m	0,89 m
15°	11,5°	5°	6,5°	10°	17,20 m	0,95 m
16°	13,5°	6°	7,5°	10°	14,40 m	1,05 m
17°	15°	7°	8°	10°	12,30 m	1,12 m
18°	16,5°	8°	8,5°	10°	10,80 m	1,18 m
19°	18°	9°	9°	10°	9,60 m	1,25 m
20°	20°	10°	10°	10°	8,60 m	1,37 m

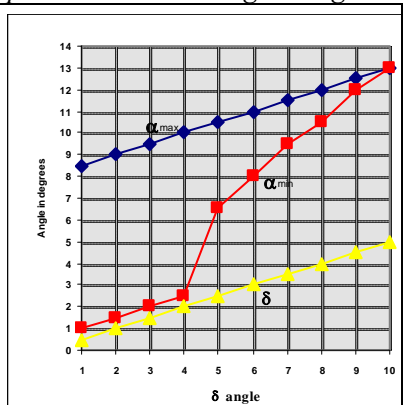


* In this table all data is for $h_a=300$ mm and curvature length (C) is 3 meters. Increasing h_a up to 400 mm and curvature length (C) to 4 meters, angles rose up to: $\alpha_{max}=22^\circ$, $\alpha_{min}=21,5^\circ$ and $\delta=12^\circ$

b) Span: 25 meters

Table 2. Values of different geometry variation parameters and their dependence concerning δ angle

α_{max}	α_{min}	δ	$\gamma_{min} = \alpha_{min} - \delta$	$\gamma_{max} = \alpha_{max} - \delta$	r_{in}	h_{ap}
8,5°	1°	0,5°	0,5°	8°	515,70 m	1,33 m
9°	1,5°	1°	0,5°	8°	257,80 m	1,35 m
9,5°	2°	1,5°	0,5°	8°	171,90 m	1,37 m
10°	2,5°	2°	0,5°	8°	128,90 m	1,39 m
10,5°	6,5°	2,5°	4°	8°	103,20 m	2,18 m
11°	8°	3°	5°	8°	86,00 m	2,42 m
11,5°	9,5°	3,5°	6°	8°	73,70 m	2,67 m
12°	10,5°	4°	6,5°	8°	64,50 m	2,80 m
12,5°	12°	4,5°	7,5°	8°	57,40 m	3,05 m
13°	13°	5°	8°	8°	51,60 m	3,19 m



* In this table all data is for $h_a=1200$ mm and curvature length (C) is 9 meters. Increasing h_a up to 1250 mm and curvature length (C) to 11 meters, angles rose up to: $\alpha_{max}=14^\circ$, $\alpha_{min}=14^\circ$ and $\delta=6^\circ$

Figure 19 Relationship between α and δ angle (25 m)

c) Span: 30 meters

Table 3. Values of different geometry variation parameters and their dependence concerning δ angle

α_{max}	α_{min}	δ	$\gamma_{min} = \alpha_{min} - \delta$	$\gamma_{max} = \alpha_{max} - \delta$	r_{in}	h_{ap}
7,5°	1°	0,5°	0,5°	7°	573,00 m	1,70 m
8°	1,5°	1°	0,5°	7°	286,50 m	1,73 m
8,5°	2°	1,5°	0,5°	7°	191,00 m	1,75 m
9°	2,5°	2°	0,5°	7°	143,30 m	1,77 m
9,5°	6,5°	2,5°	4°	7°	114,60 m	2,71 m
10°	8,5°	3°	5,5°	7°	95,50 m	3,13 m
10,5°	10°	3,5°	6,5°	7°	81,90 m	3,43 m
11°	11°	4°	7°	7°	71,70 m	3,59 m

* In this table all data is for $h_a=1550$ mm and curvature length (C) is 10 meters. Increasing h_a up to 1600 mm and curvature length (C) to 12 meters, angles rose up to: $\alpha_{max}=11^\circ$, $\alpha_{min}=10^\circ$ and $\delta=4^\circ$

4.1. Parametric design in cosmos/m

In FEM software detailed parametric analysis is conducted. Span of the girder (L), height (h_a), apex height (h_{ap}), angles α and δ were modeled parametrically. All other variables are constant value (thickness, timber class, etc). Model of the girder is composed of three different parts (as shown in Figure 18). For the purpose of design stress check (ultimate limit state), strain check (serviceability limit state) and buckling check were performed.

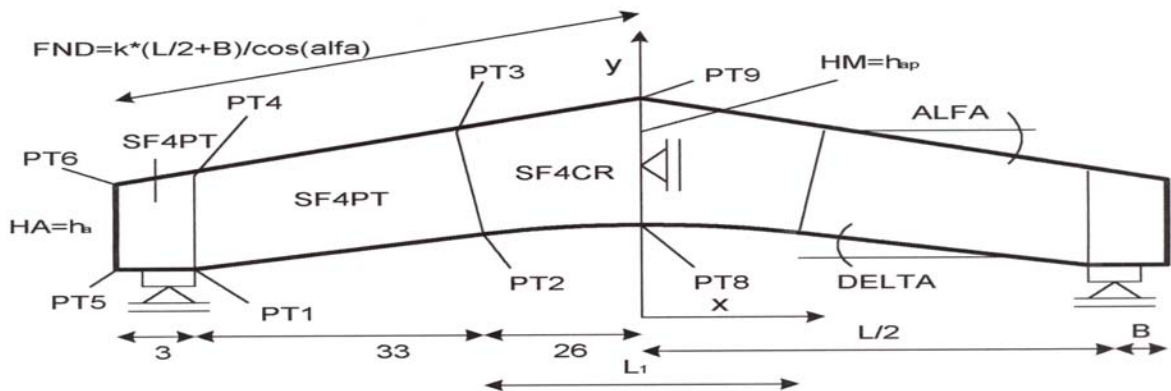


Figure 18 FE model in Cosmos/M

4.2. RESULTS OF FEM ANALYSIS

Girders with small span (10m) have both high parallel and perpendicular stresses effectively meaning that these kinds of girders are suitable for smaller spans. As span increases perpendicular stresses are becoming dominant. FEM analysis showed that large span (30 meters) girders have huge height mainly because of perpendicular stresses. On the other hand they have problems with buckling of tensile zone (as shown in figure 20). Figures 21, 22 and 23 show stresses in girders.

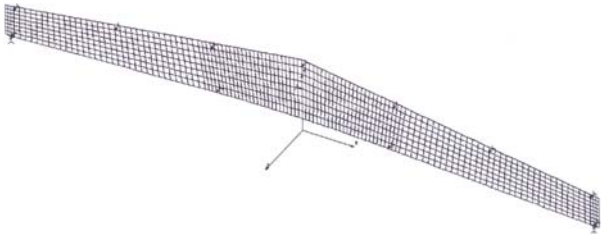


Figure 19 FE model of girder



Figure 20 Bucking of girder

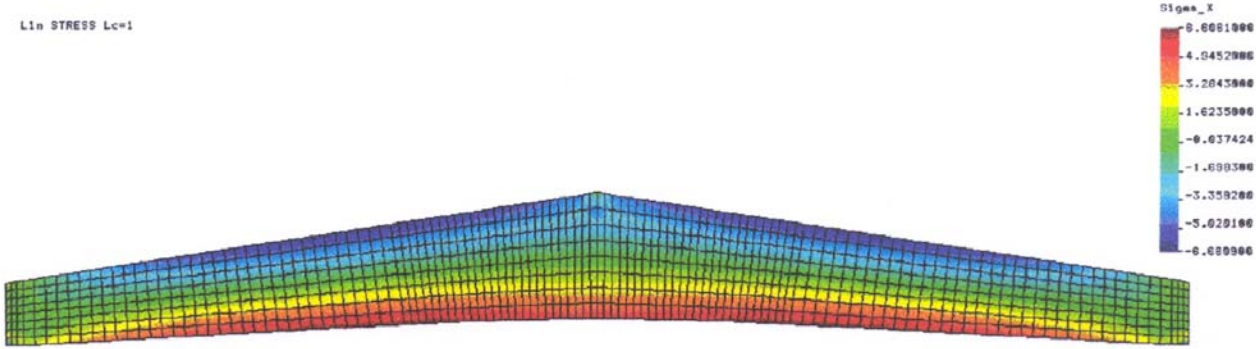


Figure 21 Stresses parallel to grain (30 meters – Max. stress 6.7 Mpa)

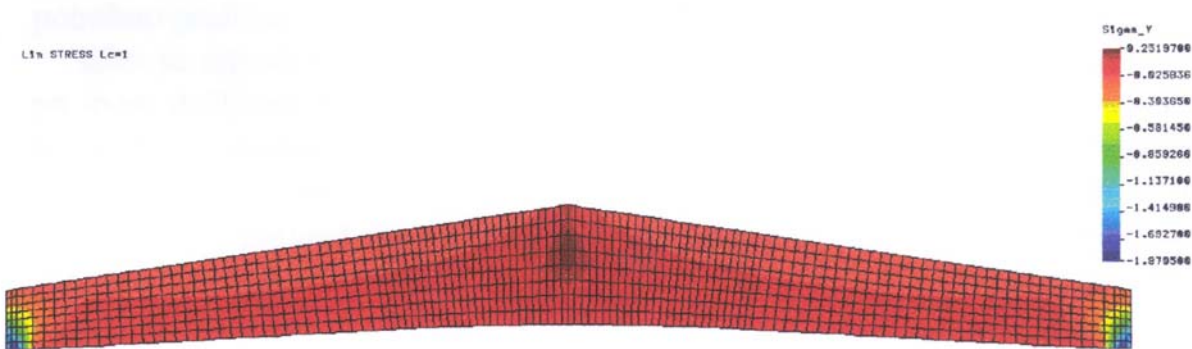


Figure 22 Stresses perpendicular to grain (30 meters – Max. stress 0.25 Mpa)

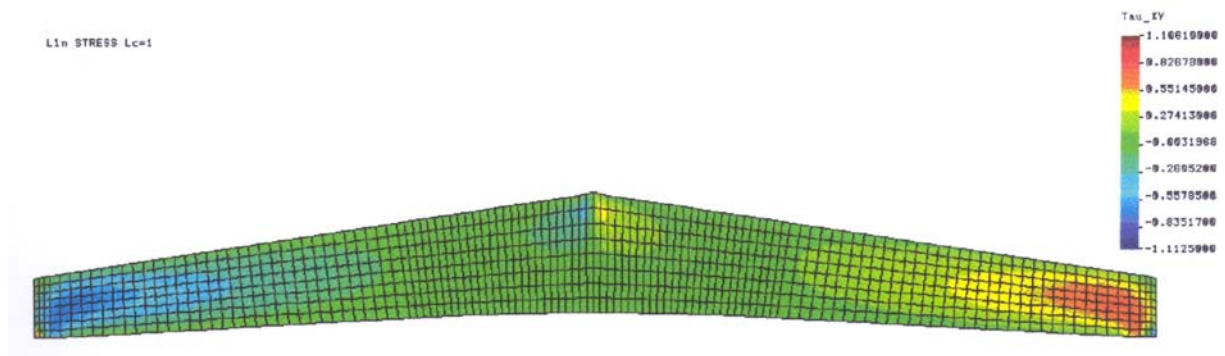


Figure 23 Shear stresses (30 meters – Max. stress 1.1 Mpa)

4.3. CONCLUSION

Biggest problem concerning glulam pitched cambered beams are tensile stresses perpendicular to grain. For spans between 10 to 20 meters tensile stresses are not crucial for design. As the span increases (20 -30 meters) these stresses are becoming more problematic and therefore crucial for design. It must be noted that stresses parallel to grain are very low, effectively meaning that beam is not fully utilized. Very small changes in angles results in big height increase. Beams with spans larger than 25 meters (25-30) have additional stability problems due to buckling in tensile zone. This analysis suggests that glulam pitched cambered beams are most efficient for span range 10-20 (maximal 25 meters).

3.5. REFERENCES

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