Size effect considerations for linear structural elements of timber

In the frame of COST E55 – Modelling of the performance of timber structures

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Abstract Material characteristics are general derived by tests under certain standardized conditions concerning climate, testing procedure, accuracy and size! In regard to mentioned requirements derived material characteristics are only valid for the reference combination of pre-formulated and standardised constraints.

Size effects in general and acc. Bažant (2004) can be divided into mechanically based sub-effects – due to energy release in partial failures and material behaviour (ductile – quasi-brittle – brittle) – and statistically based sub-effects – due to randomness of occurring properties and system behaviour (serial – serial-parallel – parallel). Current design of timber structures consider ‘size effects’ general under assumption of a perfect brittle material behaviour, often independent of loading (tension – bending – compression – shear) and independent of influences due to stochastic (e.g. EN 384, EN 1194, EN 1995-1-1, DIN 1052).

Within a finished research project at the competence centre holz.bau forschungs gmbh ‘statistical size effects’ of linear structural elements have been theoretically examined, influences of system size concerning serial linked elements and components, COV and representative statistical distribution models studied. Additional, comprehensive program by testing finger jointed construction timber (KVH$^{®}$) of various lengths in tension parallel to grain has been accomplished. Theoretical examinations clearly reflect the influence of examined distribution model, system size and COV on expectable ‘size effect’ and lead to relative simple equations to describe the ‘length effect’ on the mean and 5 %-quantile level. Practical experiments confirmed theoretically gained simplifications and reflect the predominance of stochastic in practical expectable ‘length effects’. Proposals for further regulations are given.

Key words *size effect, statistical size effect, energy release size effects, extreme value distributions, length effect, finger jointed construction timber, linear timber structures, tension parallel to grain, reference volume element*

Introduction

Mechanical characteristics of structures, particular strength properties, are in general in connection with a reference dimension. This necessity is given by the dependency of strength and size of tested specimens which can be observed by any natural and even artificial material and structure. The term ‘reference dimension’ defines herby, under consideration of observed scale, a material inherent dimension.

General ‘size effects’ describe the dependency of strength on dimension whereby a loss of strength is assumed with increasing size. In addition but not taken into account so far all mechanical and physical properties, like moisture, density, elasticity depend on size if for example the whole statistical distribution of each characteristic is considered.

The development of ‘size effects’ in our mind

In the early 1500’s Leonardo da Vinci already discussed ‘size effects’ and stated “Among cords of equal thickness the longest is the least strong”. After this time many researchers dealt with considerations about ‘size effects’, like Galileo 1638, Mariotte 1686, Griffith 1921. A comprehensive study is given in Bažant and Chen 1996. In 1926 Peirce formulated the ‘weakest link model’ by introduction of the ‘extreme value statistics’ originated by Tippett 1925. After further development of ‘extreme statistics theory’ Weibull proposed 1939 for the ‘extreme value distribution’ of strength a power law with a threshold – the 3-paramteric Weibull distribution (3pWD) of extreme value distribution type III, with threshold $x_0 = 0$ representing the 2-paramteric Weibull distribution (2pWD). The basic for the description of the ‘statistical size effect’, as a result of the randomness of strength values, has been erected. Since any material characteristic underlies certain variability between and within elements, components and systems any characteristic underlies a certain 'statistical size effect'. So far 'size effects' related with timber structures only consider the 'statistical size effect' related to strength values. Some examples within European standards are given in the next section.

So far theoretical considerations and practical tests have not been able to explain all size related effects by application of statistics theory. Bažant has been dealt with 'size effects' for decades predominantly of quasi-brittle materials like concrete. Quasi-brittle materials are characterised by stable crack growth, energy release and load redistribution of partial failures prior the achievement of the maximum load of the structure (Bažant and Chen 1996). Also wood and timber components and especially structural systems are characterised by comparable material behaviour. Further listing gives an overview of 'size effects' discussed in Bažant and Planas 1998 for concrete, enlarged with comments to relate the given knowledge for timber structures:

- ▪ <u>Boundary layer effect (wall effect)</u>
  - ⇒ As a result of heterogeneous build up of materials between boundary- and mid section layers
    - ▪ In timber such a layer is given at the outside of each structural element characterised by truncated and damaged fibers, truncated knots (most relevant on edges) whereby this layer is more or less independent of the structural size
    - ▪ Share of knot area on the surface
    - ▪ Distribution of knot sizes with high proportion of knots of small sizes which are not detected so far in visual and machine grading but have increasing influence in small cross sections
    - ▪ Influence of the position within the stem: large structural elements are predominantly cut out of the inner section of the roundwood and herby characterised by high deviation of properties within cross sections due to coexistent occurrence of juvenile and mature wood
  - ⇒ Additional stress between surface and inner layer due to heterogeneous build up of the material
- ▪ <u>Diffusion phenomena</u>
  - ⇒ Delayed transport of water, temperature and other chemicals in cyclic loading due to seasons, daily routine, etc. lead to relative affected cross sections in dependence of size
    - ▪ In timber additional orthotropic behaviour in diffusion of substances is given naturally
    - ▪ Degradation also depends on the permeability of timber and on the ration of surface vers. Volume

- Statistical size effect
  - ⇒ Caused by the randomness of material strength explainable by the ‘weakest link theory’ by attracting a chain with failure at the ‘weakest link’
    - General and also in timber applicable in case of perfect brittle material behaviour
- Fracture mechanics size effect
  - ⇒ Due to release of stored energy and redistribution of load within residual elements and components within a certain structure after partial failures
    - Of great importance in ‘quasi brittle’ materials; especially system structures are characterised by load redistribution

Additional effects are given by the transition of observable system behaviour of structures with increasing dimension. For explanation *Fig. 1* acc. Bažant and Pang 2005, Bažant 2004, Bažant et al. 2004 and Bažant and Chen 1996 reflects the transition from parallel system behaviour with length equal a reference volume element (RVE) – explainable by the normal (Gaussian) distribution – to a serial system behaviour with structural dimension $D \rightarrow \infty$:

- In case of dimension $D \rightarrow D_{RVE}$ predominant parallel acting elements lead to an ‘averaging effect’ which can be approximated by a normal distribution acc. the ‘central limit theorem’.
- With increasing dimension $D \rightarrow \infty$ the structure becomes more and more serial and can be approximated by the ‘weakest link theory’ acc. the ‘extreme value statistics’.

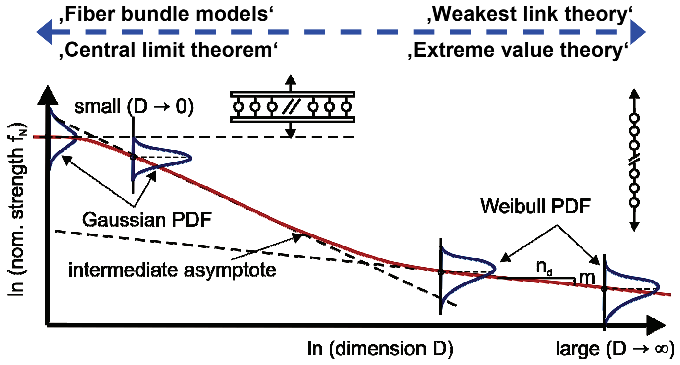

Fig. 1: Expectable ‘size effect’ of quasi-brittle materials acc. Bažant and Pang 2005, Bažant 2004, Bažant et al. 2004 and Bažant and Chen 1996: Transition from parallel- to serial acting systems with $D \rightarrow \infty$

Consideration of ‘size effects’ in current European codes and standards

Standards concerning construction timber which incorporate ‘size effect’ considerations can be subdivided into ‘standards for testing’ (e.g. EN 408), ‘standards for derivation of characteristic properties’ (e.g. EN 384) and ‘design standards’ (e.g. DIN 1052, EN 1995). All these groups linked together by regulation of reference dimensions for testing specimens, derivation of characteristic properties in relation to given reference dimensions and formulations for transformation of tested or designed structural dimensions to reference sizes. *Tab. 1* gives a short overview of so called 'size factors' of selected standards and related structural timber structures. These 'size factors' serve as strength increasing factors for structures and components with dimensions smaller the reference case. So far they have not to be considered in the other direction of larger dimensions. This lead to an overestimation of strength values in designing large structural components and too optimistic design in deep contrast to current conservative design guide lines and derived characteristics.

Tab. 1: Examples of regulated reference dimensions (D_{ref}) and derivation of 'size factors' (k_{size}) for various structural parts of timber and in dependence of related standards and loading

Structural element	**loading**	D_{ref}	k_{size}
Solid timber1) (acc. EN 384)	Tension II to grain	$l_{ref} = 9 \cdot w_{ref} = 1{,}350$ mm $w_{ref} = 150$ mm	$k_w = (150 / w)^{0.20}$
	Bending	$l_{ref} = 18 \cdot w_{ref} = 2{,}700$ mm $w_{ref} = 150$ mm	
Solid timber 2) (acc. EN 1995-1-1)	Tension II to grain	$w_{ref} = 150$ mm	$k_w = \min [(150 / w)^{0.20}; 1.30]$
	Bending		
GLT-boards 1) (acc. EN 1194)	Tension II to grain	$l_{ref} = 2{,}000$ mm $w_{ref} = 150$ mm	$k_{size} = (w / 150)^{0.10} \cdot (l / 2{,}000)^{0.10}$
	Bending	--	--
GLT 1) (acc. EN 1194)	Tension II to grain	$l_{ref} = 9 \cdot d_{ref} = 5{,}400$ mm $d_{ref} = 600$ mm $w_{ref} = 150$ mm	$k_{size} = (w / 150)^{0.05} \cdot (d / 600)^{0.10}$
	Bending	$l_{ref} = 18 \cdot d_{ref} = 10{,}800$ mm $d_{ref} = 600$ mm $w_{ref} = 150$ mm	
GLT 1) (acc. EN 1995-1-1)	Tension II to grain	$d_{ref} = 600$ mm	$k_d = \min [(600 / d)^{0.10}; 1.10]$
	Bending		
GLT 1) (acc. DIN 1052)	Tension II to grain	--	--
	Bending	$d_{ref} = 600$ mm	$k_d = \min [(600 / d)^{0.14}; 1.10]$
LVL 1) 3) (acc. EN 1995-1-1)	Tension II to grain	$l_{ref} = 3{,}000$ mm	$k_l = \min [(3{,}000 / l)^{s/2}; 1.10]$
	Bending	$d_{ref} = 300$ mm	$k_d = \min [(300 / l)^{s}; 1.20]$
LVL 1) 4) (acc. EN 14374)	Tension II to grain	$l_{ref} = 3{,}000$ mm	$k_{t,corr} = (l / 3{,}000)^{s}$ $s = COV - 0.025$
	Bending	$d_{ref} = 300$ mm	$k_{m,corr} = (d / 300)^{s}$ $s = 2 \cdot COV - 0.05$
1)	if for transformation relevant dimension is $\leq D_{ref}$		
2)	if $\rho_k \leq 700$ kg/m³		
3)	factor 's' acc. EN 14374		
4)	$COV \geq 0.05$		

The emphasise of this report and foregoing study has been taken on evaluation of the ‘statistical size effect’ of linear structures on strength values in tension parallel to the grain in dependence of the assumed representative statistical distribution model for the tension strength of the ‘reference volume elements’ (RVE’s).

Materials and Methods

In addition to introduction some theoretical considerations in relation to ‘statistical length effects’ are given further.

Theoretical considerations concerning the ‘statistical length effect’

Extreme value statistics

The theory of ‘extreme value statistics’ enable the description of the distribution of minima (min) and maxima (max) of characteristics by examination of the tails of a given statistical distribution model F(x) representing realisations x_i of a certain characteristic X of an element, component or system.

‘Extreme value statistics’ – minima $L_n(x)$

The application of this model enables the description of idealistic serial system behaviour acting like a chain (‘weakest link theory’). The strength of the system is represented by the ‘weakest link’.
By assumption of i (i = 1 ... n) realisations x_i of distributed strength values X the strength of the serial system is given by the min (x_i). If all x_i are ranked, the min (x_i) is given by x_1 with probability Pr [$x_1 \leq x$] (see [1, 2, 3, 4]).

$$x_1 = \min(x_1, x_2, ..., x_n) \rightarrow L_n(x) = \Pr[x_1 \leq x] \quad [1]$$

$$L_n(x) = \Pr[x_1 \leq x] = 1 - \Pr[x_1 > x] = 1 - \Pr[x_1 > x, x_2 > x, ..., x_n > x] \quad [2]$$

$$L_n(x) = 1 - \prod_{i=1}^{n} \Pr[x_i > x] \quad [3]$$

$$L_n(x) = 1 - (1 - F(x))^n \quad [4]$$

This leads to a decrease of strength on the mean level and reduction of dispersion expressed by $COV_{n=1}$ with increasing quantity of serial acting components n within the system.

'Extreme value statistics' – maxima $H_m(x)$

Application of this model enables the description of idealistic parallel, redundant acting elements and components within a system characterised by realisations x_j (j = 1 … m) of strength characteristic X. Herby x_m as the strongest part of ranked realisations defines the maximum potential of the attracted system (see [5, 6]).

$$x_m = \max \quad (x_1, x_2, ..., x_m) \to H_m(x) = \Pr[x_m \geq x] \tag{5}$$

$$H_m(x) = (F(x))^m \tag{6}$$

'Extreme value statistics' – some remarks

General it has to be remarked: every element, component and hence build up system structure exists of serial and parallel arranged and interacting RVE's whereby the dimension of the RVE has to be defined for every material and for every observed scale of interest, so as for wood and timber too. Concerning wood (clear wood) of spruce (picea Abies karst.) the RVE in the dimension of one fiber (tracheid) can be assumed on the micro scale. A conclusive definition of RVE for structural timber (knotty wood) on the macro scale is still missing but an essential target for the near future to enable 'size effect' considerations also in relation to 'energy release size effect' in a proper way.

Weakest link theory acc. Weibull 1939

On the basis of the 'extreme value statistic' for description of the minimum of uniform volume with equal probability of failure, given in [4], and under assumption of proper approximation of the lower tail of $L_n(x)$ by the exponential distribution Weibull derived 1939 the statistical Weibull distribution (extreme value statistics distribution type III) for characterising perfect brittle material behaviour in dependence of the volume [7] acc. Steiger 1996 (with α as location parameter and β as dispersion parameter).

$$P_f(x) = \int_{x_0}^{\infty} p_f(x) \cdot dx = 1 - \int_{x_0}^{\infty} e^{-V \cdot \left(\frac{x - x_0}{\alpha}\right)^{\beta}} \cdot dx \tag{7}$$

Based on given formula [7] the nominal strength in dependence of structural dimension can be derived acc. [8] which expresses the general consideration of 'size effects' in current standards. This formulation is independent of a predefined RVE as material inherent size. The reference dimension can be defined independent which holds true in practise in case of perfect brittle material behaviour and under the constraint of more or less equal stressed reference volumes.

$$\left(\frac{f_1}{f_2}\right) = \left(\frac{V_2}{V_1}\right)^{k_V} = \left(\frac{l_2}{l_1}\right)^{k_l} \cdot \left(\frac{b_2}{b_1}\right)^{k_b} \cdot \left(\frac{h_2}{h_1}\right)^{k_h} \tag{8}$$

Monte-Carlo-Simulations

Based on foregoing considerations and examinations for determination of representative statistical distribution models to characterise the strength of structural timber in tension parallel to grain the logarithmic normal distribution (LND), and due to the 'weakest link theory' acc. Weibull 1939 the Weibull distribution has been taken as distribution models to characterise the tension strength of virtual 'reference volumes' (RVE's). Due to lack of close solvable integral of LND Monte-Carlo-simulation technique has been applied to examine the 'size effect' of serial systems in dependence of the basis strength distribution model, the dispersion ($COV_{n=1}$) and the system size (n). Further more key statistic figures like mean, coefficient of variation (COV), skewness, kurtosis, 5 %-quantile and others of virtual strength values of generated systems have examined under application of rank statistic and fitted distribution models like normal distribution (ND), 2-and 3-parametric log. normal distribution (2p-3pLND) and 2- and 3-parametric Weibull distribution (2p-3pWD). Basic strength data sets for RVE's have been generated based on acc. statistical distribution model and linked parameters inverse transformed uniform distributed random numbers. Systems have been generated randomly acc. required system size n. The system strength value $f_{t,0,n}$ has been calculated by taken the minimum of all included strength realisations $f_{t,0,i}$ (i = 1 … n) acc. the 'weakest link theory' (WLT) in regard to 'extreme value statistics'.

Practical experiments for the identification of 'length effects' in tension parallel to the grain

In addition to theoretical examinations practical tests have been accomplished on finger jointed construction timber KVH$^{®}$ of spruce (picea Abies karts.), provenience Middle Europe, graded visual to S10+ acc. DIN 4074 (sample without reject and S7 but with S10 and better) with finger joint profile 20 / 5 acc. EN 385.
To enable a representative sample KVH$^{®}$/I, rods of planed cross section l / w / d = 18,000 / 160 / 60 mm have been taken from ongoing production and divided into 3 sub-samples as given in *Tab. 2*.
The tension tests on KVH$^{®}$/I-A have been accomplished by Jeitler et al. 2007 at the competence center holz.bau forschungs gmbh acc. EN 408 with free testing length $l_0 = 9 \cdot w = 9 \cdot 160 = 1{,}440$ mm. Tension tests on sub-samples KVH$^{®}$/I-B and KVH$^{®}$/I-C have been also carried out by Jeitler et al. 2007 at the 'proof loading' testing device (described in Katzengruber et al. 2006 a, b) at the company Holzindustrie Preding GmbH by use of adjusted grips.

Tab. 2: Sub-samples A, B, C of sample KVH$^{®}$/I: quantities, length and free testing length of tension test specimens tested parallel to the grain (free length between grips)

sub-samples	quantity [--]	length l [mm]	free testing length l_0 [mm]
KVH$^{®}$/I-A	98 #	2,540	1,440
KVH$^{®}$/I-B	51 #	9,000	7,982
KVH$^{®}$/I-C	42 #	18,000	17,222

All tests have been carried out by around u = 12 % moisture content, by determination of density and recording the failure characteristics.

Results of practical tests and simulations

Theoretical results gained from simulations

Further figures reflect some main outcomes of simulations in dependence of supposed statistical distribution model for representation of RVE's tension strength $f_{t,0}$ parallel to the grain.

Theoretical results and plots on the basis of LND-RVE's

Fig. 2 contains results of 'length effect' factors for tension strength of virtually generated serial systems. The k_{length}-factors have been calculated under application of formula [8] based on 5 %-quantiles derived acc. given statistical distribution models and based on empirical distributions (rank analysis) (see Brandner et al. 2007). Herby the derived k_{length}-factors converge asymptotic to around k_{length} = 0.90, represented by the 2pWD, with increasing n. By ND represented system strength values lead to the highest deviation of asymptotic value in case of small sample sizes, whereas fitting of LND reflect the same trend qualitative and more or less quantitative as the k_{lentgh} based on rank analysis of empirical distribution model (empD). The application of 2pWD for calculation of the 5 %-quantiles of system strength, as the basis for carried out analysis, reflect a flat line.

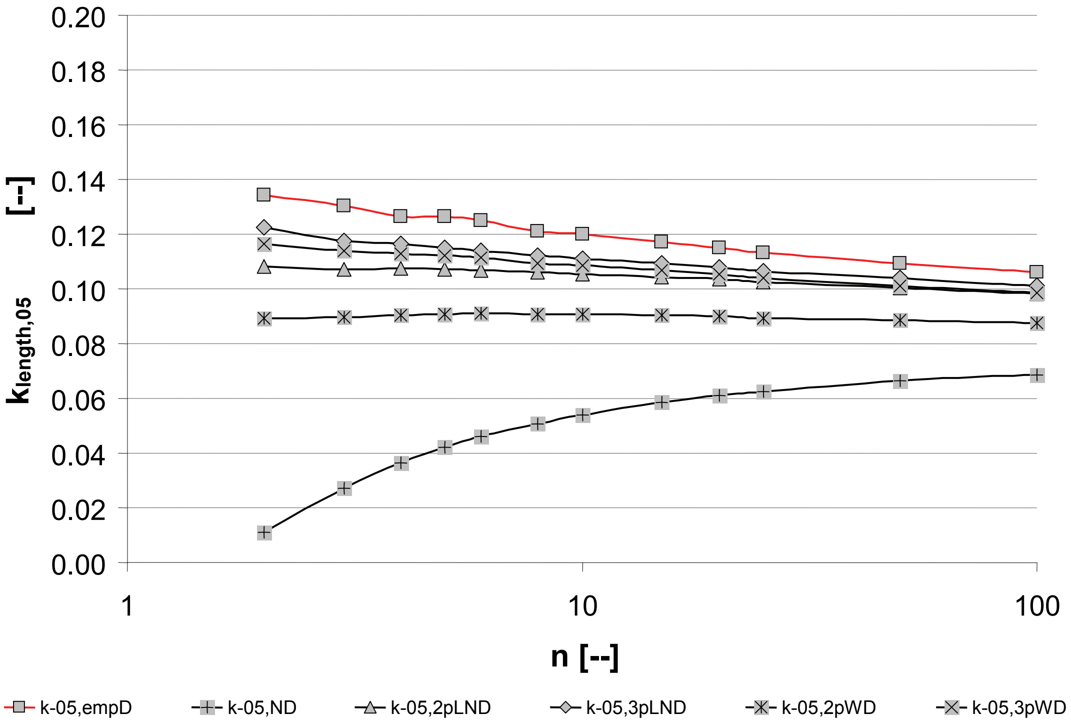

Fig. 2: Variation of k_{length} on the level of the 5 %-quantile acc. assumed statistical distribution models to characterise the simulated system strength values, under assumption of LND-RVE's with $f_{mean,n=1}$ = 50 N/mm² and COV-$f_{n=1}$ = 30 %

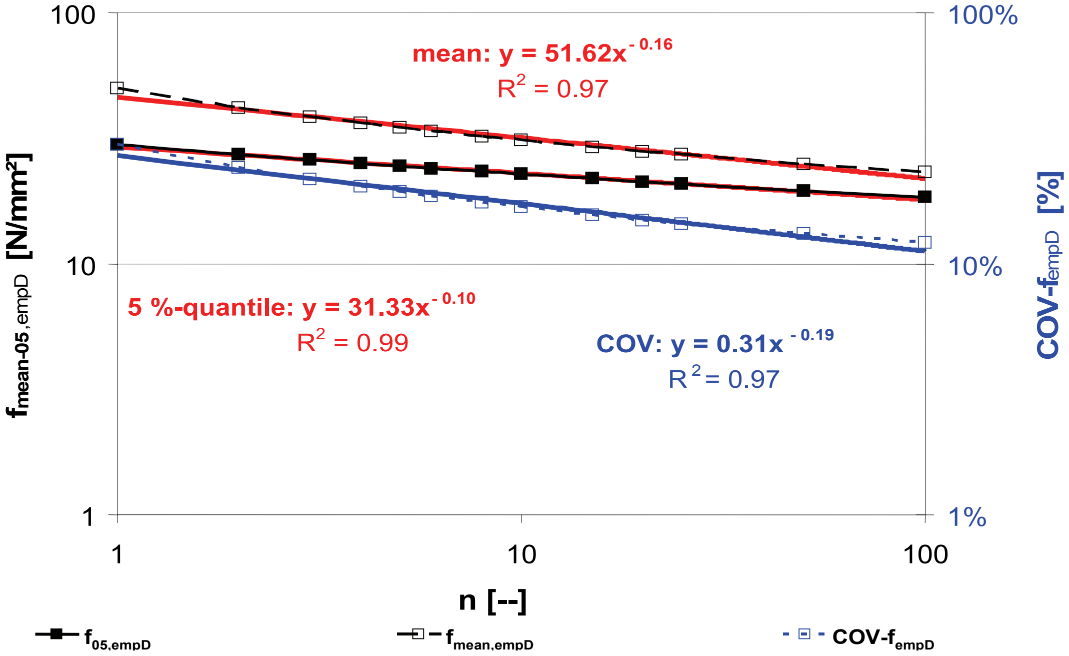

Fig. 3: *'Length effect' on mean-, 5 %-quantile- and COV-basis of serial systems, examined based on empirical distributions empD (rank analysis) under assumption of LND-RVE's with $f_{mean,n=1}$ = 50 N/mm² and COV-$f_{n=1}$ = 30 %*

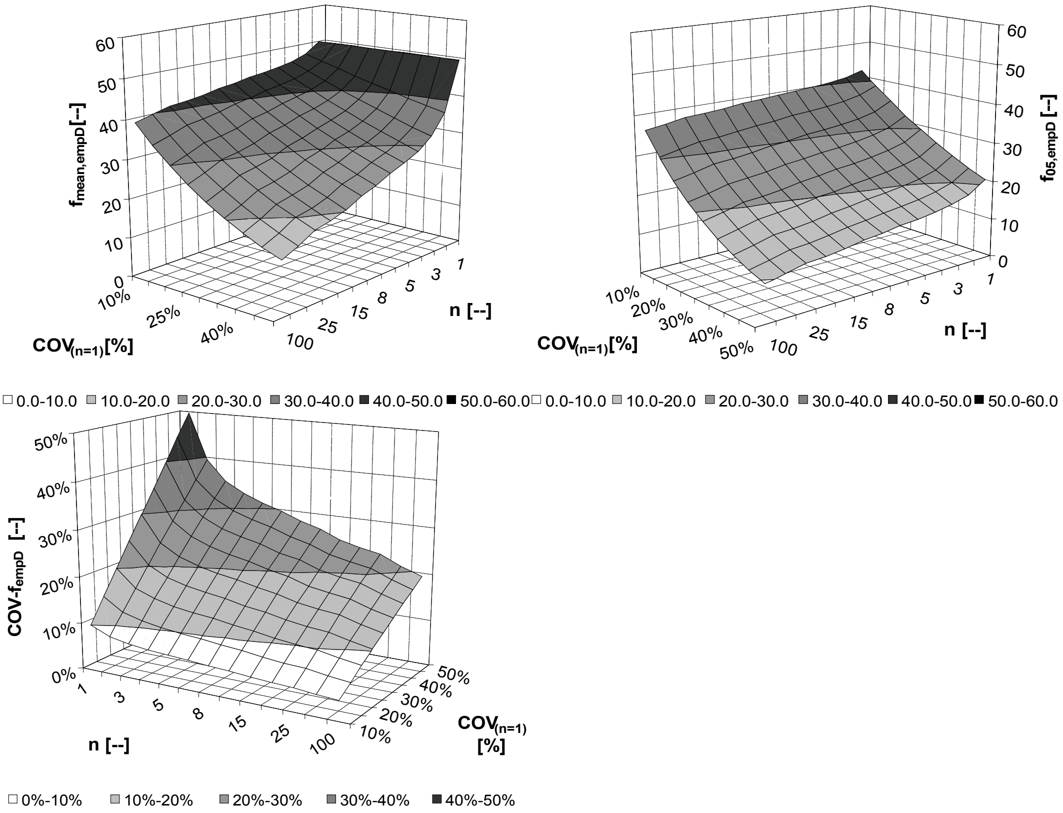

Fig. 4: *Simulation results of strength values of serial systems gained from empirical distribution empD (rank analysis) based on LND-RVE's with $f_{mean,n=1}$ = 50 N/mm² and COV-$f_{n=1}$ = 30 %: dependency of $f_{mean,empD}$ of $COV_{n=1}$ and n (left above); dependency of $f_{05,empD}$ of $COV_{n=1}$ and n (right above); dependency of COV-f_{empD} of $COV_{n=1}$ and n (left below)*

Fig. 3, as example with COV-$f_{t,0,LND,n=1}$ = 30 %, points out the dependency of the system size on $f_{t,0,l,mean,n}$, $f_{t,0,l,05,n}$ and COV-$f_{t,0,n}$ in logarithmic scale and enables the calculation of k_{length}-factors based on fitted power functions. The decrease of COV-$f_{t,0,l,n}$ in combination with $f_{t,0,l,mean,n}$ lead to a slightly reduction of $f_{t,0,l,05,n}$ ($k_{length,mean}$ = 0.16, $k_{length,COV}$ = 0.19, $k_{length,05}$ = 0.10). The influence of COV-$f_{t,0,l,n=1}$ of RVE's and the system size n on expectable statistics of system strength $f_{t,0,n}$ of serial systems is given in *Fig. 4*.

Theoretical results and plots on basis of WD to represent strength of RVE's

As already shown for LND-RVE's also examinations on WD-RVE's have been accomplished. *Fig. 5* reproduces the connection between the system size n and the statistic of system strength values on mean-, 5 %-quantile- and COV-level. In contrast to *Fig. 3* of LND-RVE's the 'size effect' on COV-$f_{t,0,n}$ is equal to zero based on the fact that the WD already incorporates the decrease of COV with increasing n as 'extreme value distribution' type III. Due to decreasing $f_{t,0,mean,n}$ also $f_{t,0,05,n}$ decreases with the same negative gradient in logarithmic scale.

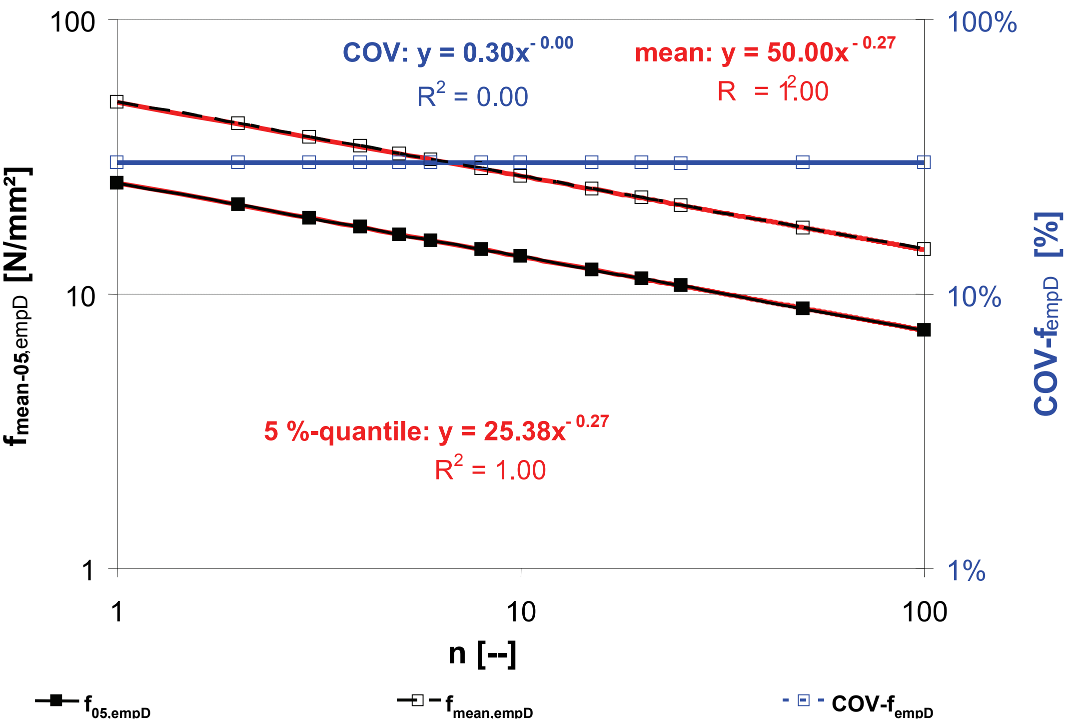

Fig. 5: *'Length effect' on mean-, 5 %-quantile- and COV-basis of serial systems, examined based on empirical distributions empD (rank analysis) under assumption of WD-RVE's with $f_{mean,n=1}$ = 50 N/mm² and COV-$f_{n=1}$ = 30 %*

Test results

Tab. 3 includes the statistics of density and tension strength of the sample KVH®/I-A. On the basis of given characteristic values of $f_{t,0,k,EN 14358}$ = 14.7 N/mm² and $\rho_{k,EN 384}$ = 374 kg/m³ the requirements for the strength class ≥ C24 acc. EN 338 for grading class S10+ acc. DIN 4074 are fulfilled. Analysis of the distribution of tension strength values by examination of ND, 2p-3pLND and 2p-3pWD lead to best representation by 2pLND, predominantly at the lower tail of the empirical cumulative distribution function (empD).

Tab. 3: Key statistic figures of density and tension strength of sub-sample KVH®/I-A

	ρ [kg/m³]	$f_{t,0}$ [N/mm²]
quantity	98 #	98 #
mean	440	27.5
COV	9.1 %	31.7 %
5 %-quantile (ND)	374	--
5 %-quantile (LND)	--	15.2
charact. value acc. EN 384	374	--
charact. value acc. EN 14358	--	14.7

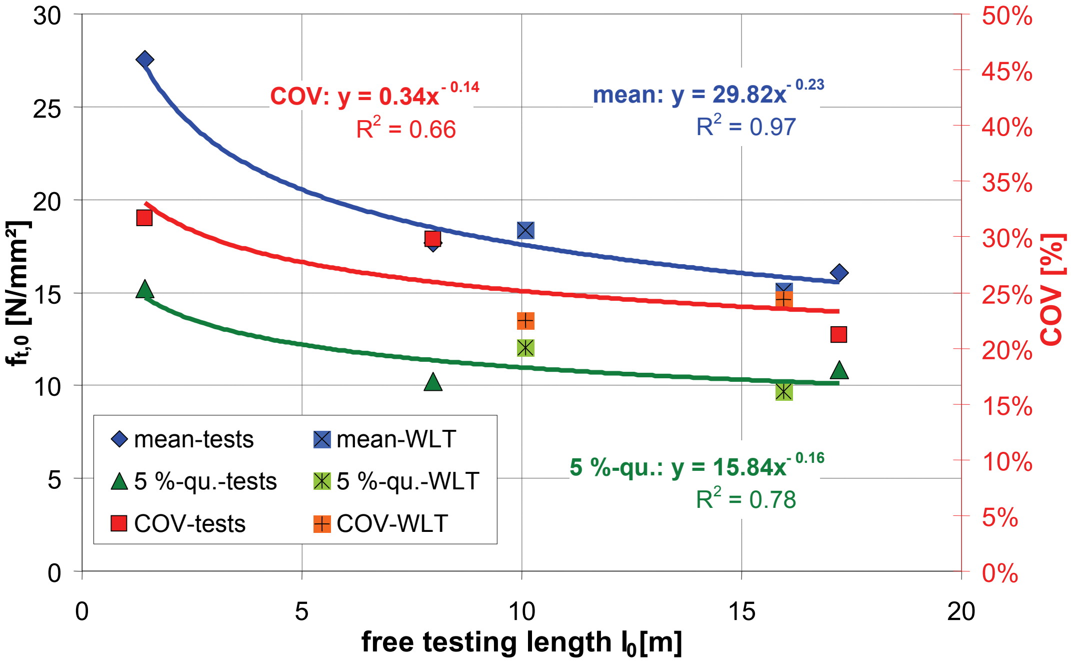

Fig. 6: Statistics tension strength of test results and simulated results on KVH® on the mean-, 5 %-quantile- and COV-level, in dependence of free testing length: sim. data sets have been gained by application of 'weakest link theory' (WLT) on tests results

The mean ($f_{t,0,mean,n}$), 5 %-quantile ($f_{t,0,05,n}$) and COV-$f_{t,0,n}$ of practical tested samples in relation to the applied free testing length are given in *Fig. 6*. The fitted power functions with power equal the k_{length}-factor acc. the WLT give good representation of limited data sets and lead to $k_{length,mean}$ = 0.23, $k_{length,COV}$ = 0.14 and $k_{length,05}$ = 0.16 in regard to COV-$f_{t,0,KVH/I-A}$ = 31.7 %.
Based on test results of KVH®/I-A and KVH®/I-B virtual random systems KVH®/I-Av and KVH®/I-Bv have been build up. The system strength values have been calculated based on WLT.

These additional data sets enlarge the practical gained data points in a proper way and lead only to negligible changes of the powers of fitted power functions below 1 / 100. On this way, the applicability of the 'weakest link theory' for the explanation of practical 'length effects' of timber structures in tension parallel to the grain can be confirmed.

Discussion

Additional sample KVH®/II for further examinations

Within the research project P03 qm_online III of the competence center holz.bau forschungs gmbh a sample KVH®/II of ungraded KVH®/II of spruce, Middle Europe, with planed cross section w / d = 160 / 60 mm has been tested in tension parallel to the grain acc. EN 408 with free testing length $l_0 = 9 \cdot w = 1{,}800$ mm. Some characteristic values are given in *Tab. 4*. Analysis concerning the representative statistical distribution model lead to 2pLND.
Based on extensive data set systems, by varying n, have been generated by random grouping of strength values. The system strength values have been calculated acc. WLT. Gained k_{length} factors have been included in further examinations.

Tab. 4: Key statistic figures of density, modulus of elasticity and tension strength of sample KVH®/II

	ρ **[kg/m³]**	**$E_{t,0}$** **[N/mm²]**	**$f_{t,0}$** **[N/mm²]**
quantity	220 #	220 #	220 #
mean	441	10,920	28.1
COV	10.0 %	28.0 %	44.8 %
5 %-quantile (ND)	366	--	--
5 %-quantile (LND)	--	--	11.8

Comparison of test results with virtual systems based on LND-RVE's

Fig. 7 reflects the k_{length}-factors on the mean-, 5 %-quantile- and COV-level, based on simulation data analysed by empD, in dependence of the main influencing parameter COV-$f_{t,0,n=1}$. Additional standard deviation of characteristic values is given. As constraint of the relationship k_{length} vers. $COV_{n=1}$ the trivial solution of $k_{length,COV \to 0} \to 0$ has to be considered. A comparison with the test data of samples KVH®/I and KVH®/II gives good agreement with the simulated data sets and reflects first the high influence of $COV_{n=1}$ on expectable k_{length}-factors on the mean- and 5 %-quantile level of tension strength, and second the possibility to handle the 'length effect' of linear construction timber by application of the WLT on the basis of the representative statistical distribution model to characterise the strength of the reference volumes (in practical tests this is conform to KVH®/I-A and KVH®/II).

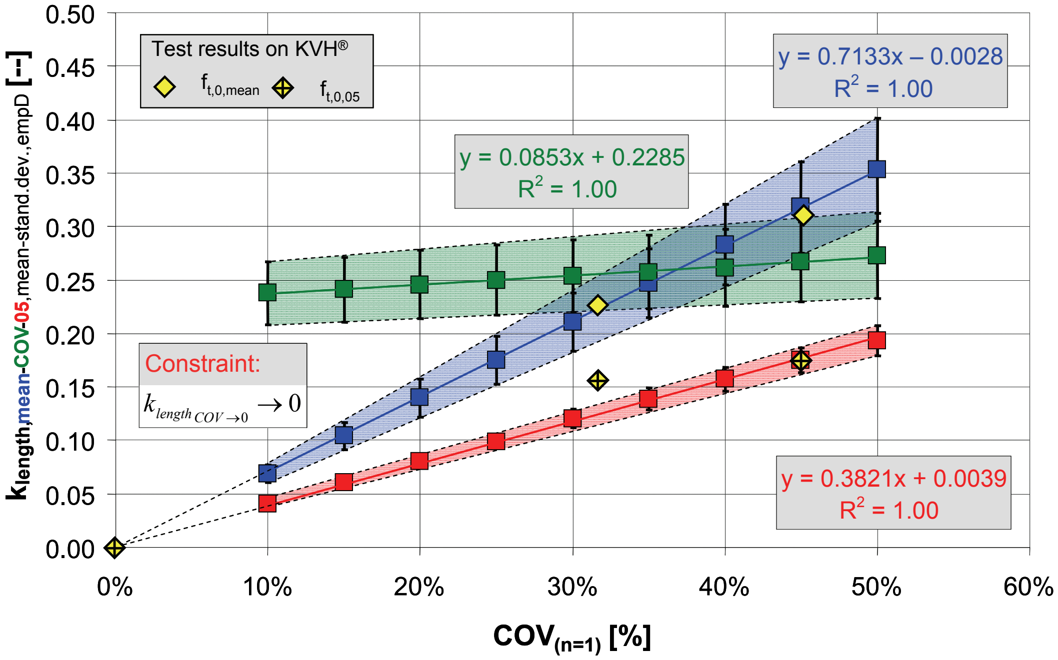

Fig. 7: Comparison of test results of series KVH®/I and KVH®/II with simulated data based on assumed LND representing RVE's tension strength

Based on derived fitted linear regression models and under consideration of constraint of $k_{length,COV \rightarrow 0} \rightarrow 0$ some simplifications concerning regulation of k_{length} can be made and given in [9, 10].

$$k_{length,mean,LND} \approx 0.7 \cdot COV_{n=1} \qquad [9]$$

$$k_{length,05,LND} \approx 0.4 \cdot COV_{n=1} = 0.6 \cdot k_{length,mean,LND} \qquad [10]$$

Comparison of test results with virtual systems based on WD-RVE's

As already given for LND-RVE's in *Fig. 7* is for WD-RVE's shown in *Fig. 8* with the same constraint based on the trivial solution $k_{length,COV \rightarrow 0} \rightarrow 0$. As mentioned before the 'statistical length effect' of serial systems build up of WD-RVE's only affects the strength data on the mean- and 5 %-quantile basis. The COV-$f_{t,0,n}$ is not affected because the reduction of COV is already incorporated in the Weibull distribution as one representative of the 'extreme value distributions' family.

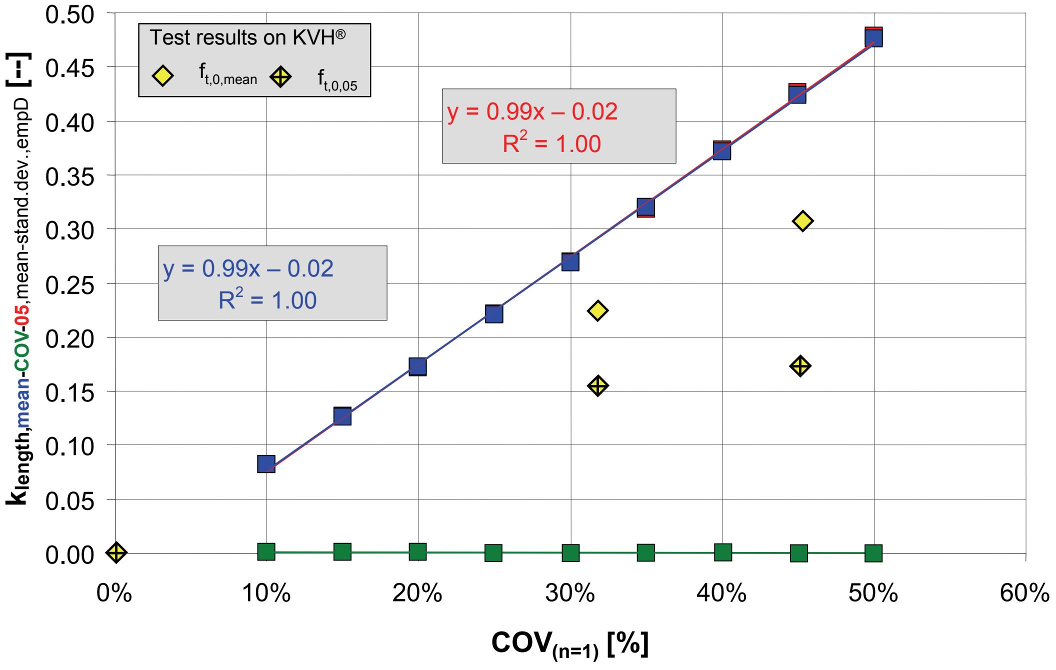

Fig. 8: Comparison of test results of series KVH®/I and KVH®/II with simulated data based on assumed WD representing RVE's tension strength

The comparison with practical data sets gives no agreement, not on mean-, nor on the 5 %-quantile level. This has been expected since the LND, not the WD has been found to represent the tension strength data sets best. Nevertheless for WD-data sets a simple formulation can be derived to calculate the k_{length}-factor in dependence of the COV-$f_{t,0,n=1}$ as given in [11].

$$k_{length,05,WD} \approx 1.01 \cdot COV_{n=1}^{\;1.1} \qquad [11]$$

Conclusions

The 'statistical size effect' of linear structures of timber has been examined based on simulations and by verification of simulation data by practical tests on finger jointed construction timber KVH®. Some comments concerning the 'weakest link theory' acc. Weibull 1939 and the 'extreme value theory' in general are given. In addition current thoughts concerning sub-effects of 'size effects' based on extensive research work of Bažant and Pang 2005, Bažant 2004, Bažant et al. 2004, Bažant and Planas 1998 and Bažant and Chen 1996 on concrete are discussed and transferred to timber constructions.

Based on the results and the definition of the 'statistical size effect' due to the randomness of strength, the $COV_{n=1}$ has been determined as the predominant influence on the k_{size} factors and on k_{length} of linear timber structures in tension parallel to the grain in particular.

The comparison of simulated data sets and practical derived characteristics gave very good agreement in case of LND-RVE's, whereby it has to be mentioned, that the LND best represented the tension strength values of the practical derived tests of sample KVH®/I-A and KVH®/II.

Some modelling of k_{length}-factors in dependence of COV-$f_{t,0,n=1}$ and under consideration of the constraint of $k_{length,COV\rightarrow 0} \rightarrow 0$ enabled derivation of simple formulations to take into account the k_{length} of linear timber structures as a function of the representative statistical distribution model for representation of the tension strength values of the reference dimension and the coefficient of variation ($k_{length} \rightarrow f\,\{\text{DM-RVE}, COV_{n=1}\}$).

Acc. current regulations and in regard to the standard EN 14374 of LVL a proposal is given for further regulation of k_{length}-factors for linear timber structures in tension parallel to the grain like for solid timber elements, finger jointed construction timber, GLT-lamellas and even for GLT and LVL. The predominance of LND as representative statistical distribution model of tension strength values parallel to the grain has already been proven even for boards for GLT-production in Brandner and Schickhofer 2007. The proposal is given for the distribution model (DM) $LND_{n=1}$ and in dependence of $COV_{n=1}$ (see [12, 13, 14]).

$$k_{length,t} = \left(\frac{l}{l_{ref}}\right)^{\alpha_t} \rightarrow \left(\frac{l}{9 \cdot b_{ref}}\right)^{\alpha_t} = \left(\frac{l}{9 \cdot 150}\right)^{\alpha_t} = \left(\frac{l}{1350}\right)^{\alpha_t} \qquad [12]$$

$$\alpha \rightarrow f\{DM_{n=1}, COV_{n=1}\} \qquad [13]$$

$$\alpha_{t,LND} = 0.4 \cdot COV_{n=1} \qquad [14]$$

On the basis of above proposal following $k_{length,t}$-factors α_t in dependence of $COV_{n=1}$ are given in *Tab. 5*.

Tab. 5: $k_{length,t}$-factors α_t in dependence of $COV_{n=1}$

$COV_{n=1}$	$k_{length,t}$-factor α_t	$COV_{n=1}$	$k_{length,t}$-factor α_t
5 %	0.025	25 %	0.100
10 %	0.040	30 %	0.120
15 %	0.060	35 %	0.140
20 %	0.080	40 %	0.160

In reference to the range of 'sub-size effects', discussed at the beginning of the paper, and in reference of *Fig. 1* the application of WLT for representation not only of 'statistical size effects' but even for 'length effects' in linear timber structures in tension parallel to the grain, it is assumed that the dimension of RVE's is, compared to tested and in field applied dimensions, small enough that a predominant serial acting system behaviour can be practically assumed. Further more the behaviour of timber in tension parallel to the grain can be assumed as nearly brittle.

The consideration of 'size effects' in derivation of characteristic properties in laboratory tests as well as in the design of timber structures is essential, for structures smaller the reference size and even more for structures bigger the reference dimensions.

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