System Reliability – Ductility and Redundancy

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Content

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- System Reliability
- System Reliability – Ductility & Redundancy
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- Future Work

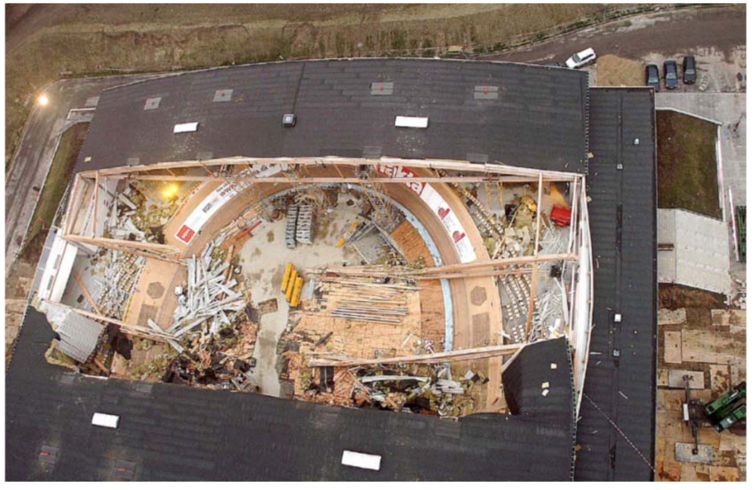
COST E55:WG3 – Objectives

- Characterisation of multi-scale variability in timber structures
- Analysis of system effects for several types of timber structures
- Qualification of robustness as a characteristic of timber structures
- Establishing a framework for reliability based design and assessment of timber structural systems based on these considerations.

COST E55:WG3 – Objectives

- Ballerup arena
  Copenhagen, Denmark

  2 out of 12 main trusses collapsed

  Ice skating arena
  Bad Reichenhall, Germany

  Total collapse

- Hazards (design error, unforeseen incidents, …): correlated / uncorrelated for different elements?
- Connection between main trusses: strong / weak ?
- Brittle / ductile failure type?

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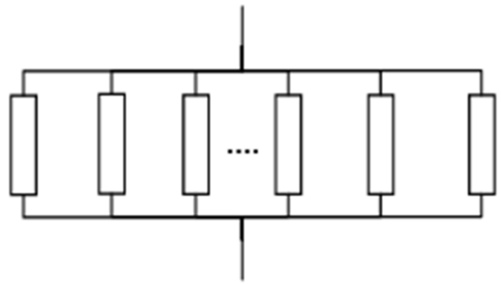
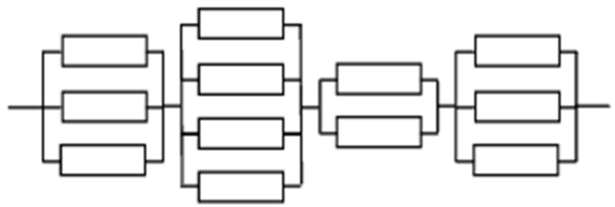
System Reliability

- Series system

- Parallel system



- Hybrid system



System Reliability

- Element

$$P_f = P\left(M \le 0\right) = P\left(R(\overline{X}) - S(\overline{X}) \le 0\right) = P\left(g(\overline{X}) \le 0\right)$$

$$g(\overline{X}) = 1 - \left(\frac{\sum_i S_{t,i}}{(z_{d,A} \cdot R_{t,0})} + \frac{\sum_i S_{m,i}}{z_{d,M} \cdot R_m} \right) \cdot X_M = 0$$

- Series system

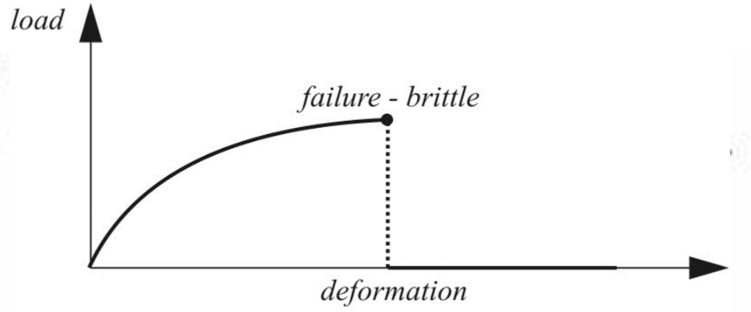
$$P_f^S = P\left(\bigcup_{i=1}^{m} \left\{ M_i \le 0 \right\} \right) = P\left(\bigcup_{i=1}^{m} \left\{ g_i(\mathbf{X}) \le 0 \right\} \right) = P\left(\bigcup_{i=1}^{m} \left\{ g_i\left(\mathbf{T(U)}\right) \le 0 \right\} \right) \approx \Phi(-\beta^s)$$

- Parallel system

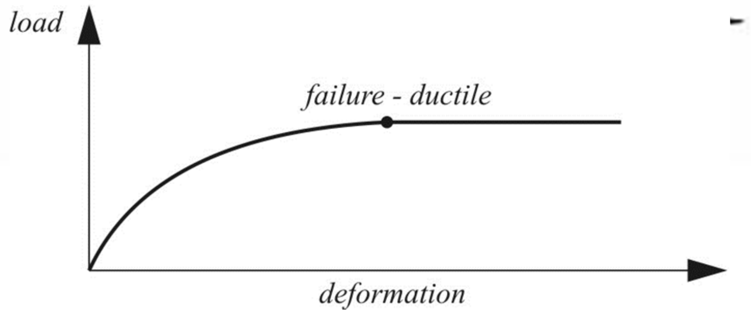
$$P_f^P = P\left(\bigcap_{i=1}^{m} \left\{ M_i \le 0 \right\} \right) = P\left(\bigcap_{i=1}^{m} \left\{ g_i(X) \le 0 \right\} \right) \approx \Phi(-\beta^p)$$

System Reliability – Ductility & Redundancy

- The behavior of structural failure modes after failure is important for assessing the safety (Robustness) of a structural system. Two extreme failure modes are:
  - brittle failure mode



  - ductile failure mode



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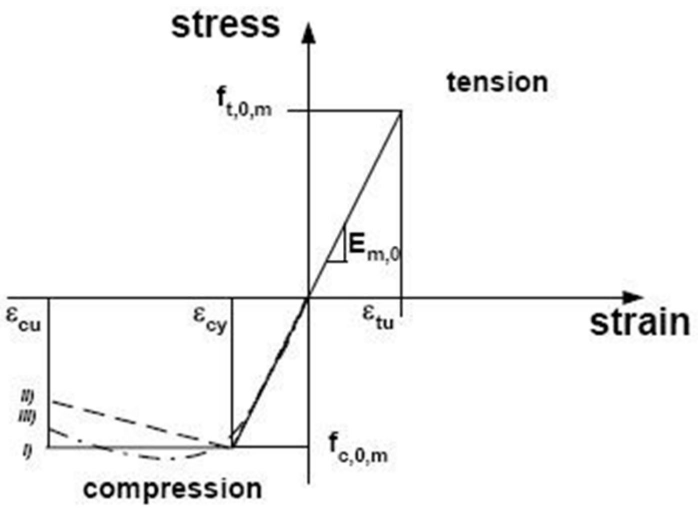
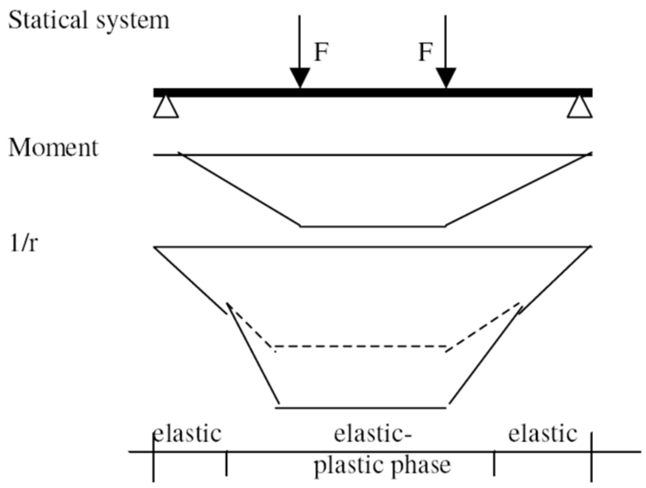
Ductile/Brittle Material Behaviour

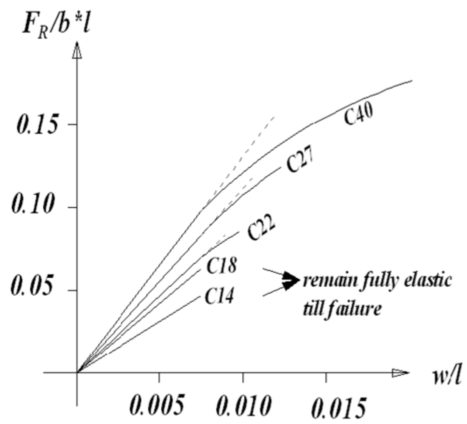
- In general timber is considered to be a brittle material, because failure occurs suddenly, without any warning. This can be considered as an obstacle when comparing to other materials like steel. It has none or a very little ductility in the tensile area, while in compressive area linear elastic-plastic behaviour can be assumed



Ductile/Brittle Material Behaviour

- The better grade timbers do exhibit some ductile behavior, but the deviation from the straight line of the force-deflection-relationship is a minimum. Only when very high grade timber (C35 or C40) is used can a marked deviation from the straight line be expected.





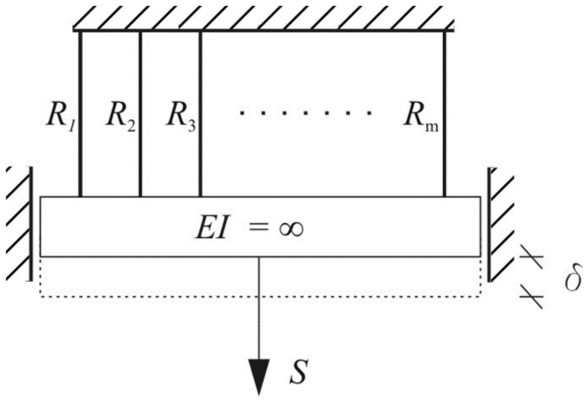
Brunner, M. *On the Compressive Strength of Timber in Bending*. 2004.

System Reliability – Ductility & Redundancy

- Simplified system modeling of ductile/brittle structural system

  - Parallel system with *m* elements
  - Perfect ductile / brittle elements
  - Load distribution after element failure

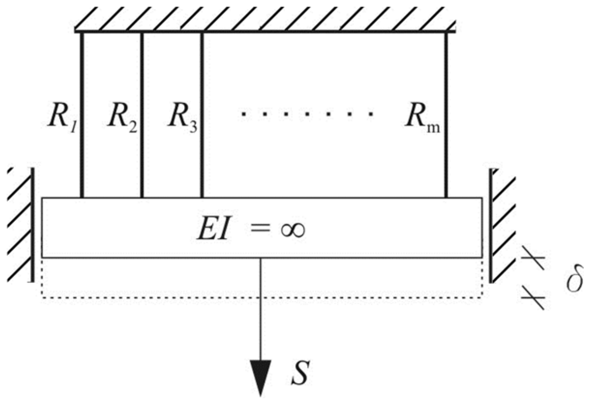
$$F_{sys} = \max_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \le 0\right) = \bigcap_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \le 0\right)$$



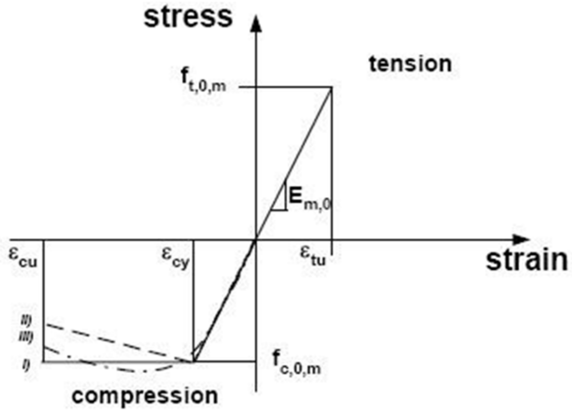
- ✓ Daniels, H.E. *The statistical theory of the strength of bundles of threads, Part I.* in *Proceedings of the Royal Society*. 1945
- ✓ Frangopol D.M. and Curley J.P., *Effects of damage and redundancy on structural reliability.* Journal of Structural Engineering, 1987. **113**(7): p. 1533-1549.
- ✓ Gollwitzer, S. and R. Rackwitz, *On the reliability of Daniels systems.* Structural Safety, 1990. **7**: p. 229-243.
- ✓ Baker, J.W., M. Schubert, and M.H. Faber, *On the assessment of robustness.* Journal of Structural Safety., 2007. **30**(3): p. 253-267.

Example

➢ Simplified system modeling of ductile timber structural system



$$F_{sys} = \max_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \leq 0\right) = \bigcap_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \leq 0\right)$$



Ductility measure $D = \frac{u_f}{u_y} = 1,2,..,4$

- Parallel system with *m* elements
- Perfect ductile / brittle
- Load distribution after element failure
- Dead load (G) and live load (Q) Normal and Gumbel distributed
- Resistances (R) Lognormal distributed
- Limit state function

$$S = (1-\alpha)G + \alpha Q$$

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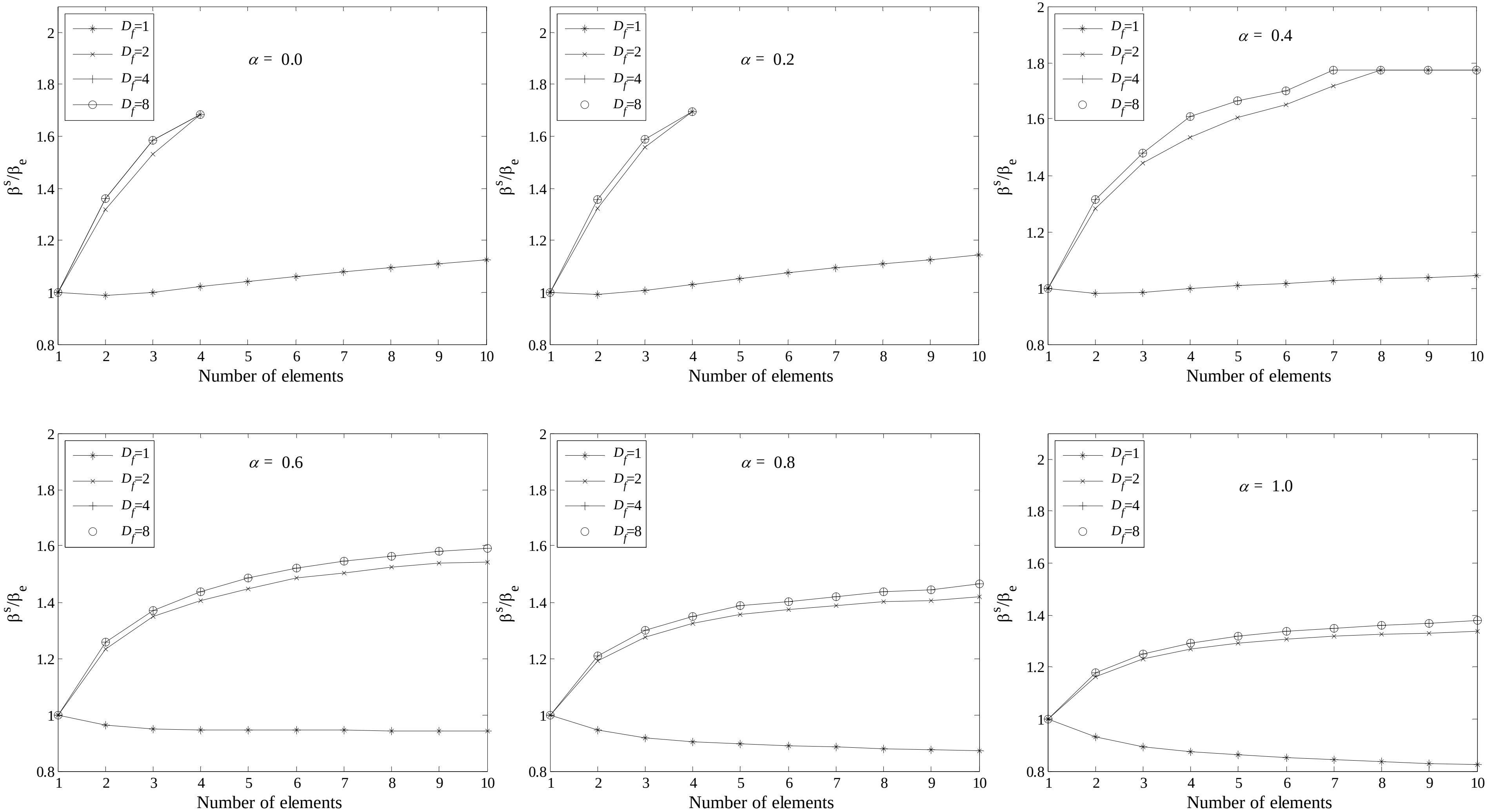
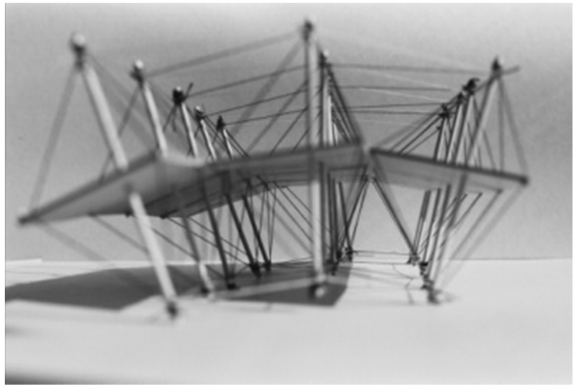
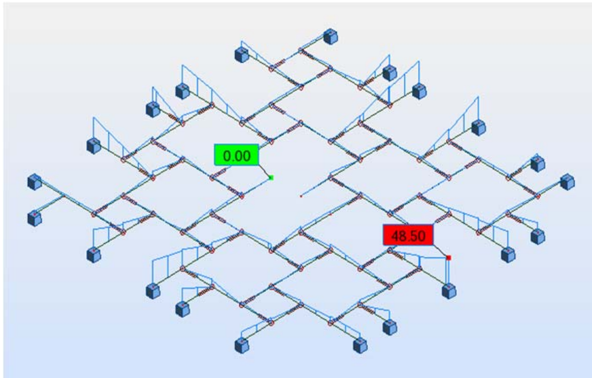
Results



Future Work (on-going research)

- The ductility effect could be taken into account for improving the robustness of timber structures
- Robustness of Long Span Reciprocal Timber Structures, IASS 2011
- CLT used for plate tensegrity , IASS 2011





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for
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