Reliability Reliability-based robustness analysis for a Croatian sports hall

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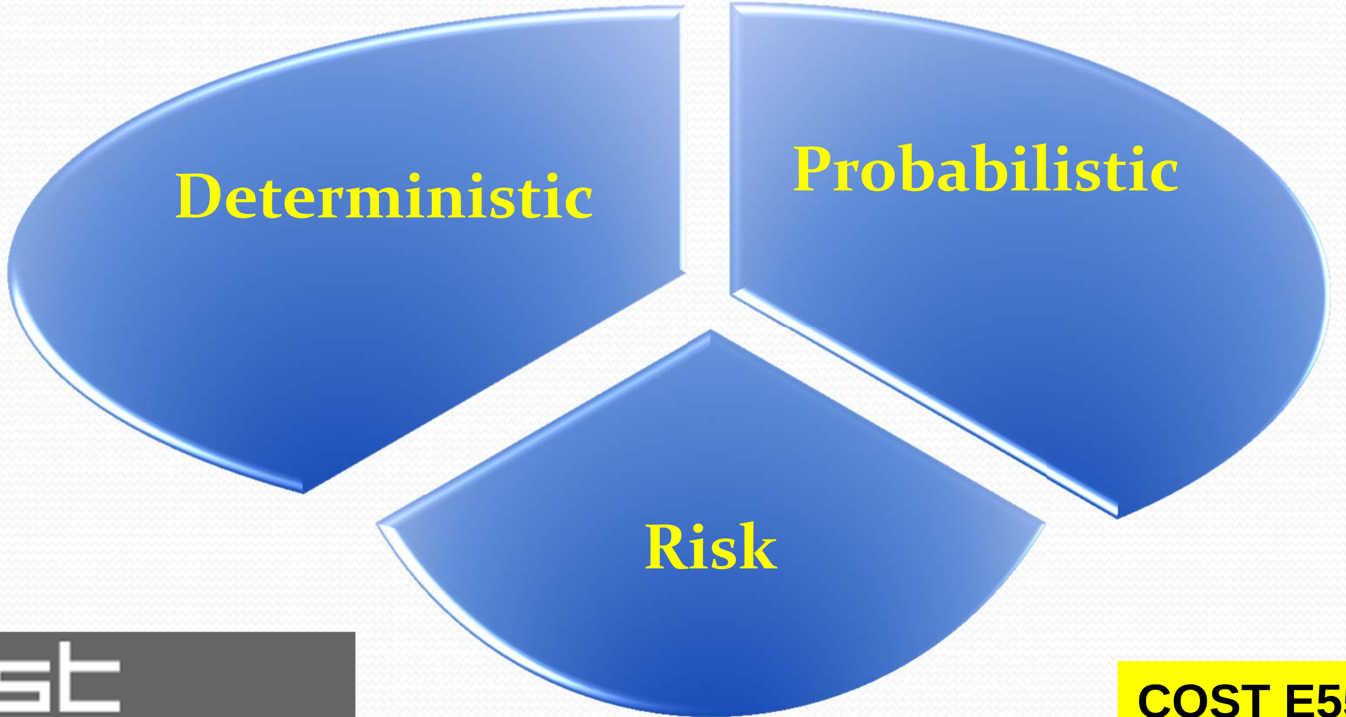
Introduction

Different definitions of robustness



Introduction

PROBABILISTIC (RELIABILITY BASED) DEFINITIONS

Frangopol and Curley - probabilistic structural redundancy index (RI):

$$RI = \frac{P_{f(dmg)} - P_{f(sys)}}{P_{f(sys)}}$$

high robustness : $RI \to 0$

low robustness : $RI \to \infty$

redundancy factor

$$\beta_R = \frac{\beta_{\text{int act}}}{\beta_{\text{int act}} - \beta_{damaged}}$$

high robustness : $\beta_R \to \infty$

low robustness : $\beta_R \to 0$

The vulnerability (V) of a system (Lind):

$$V = \frac{P(r_d, S)}{P(r_0, S)}$$

Very hard to classify structures according to these robusteness indices – range $[0, \infty >$

Introduction

In this article a new robustness index is proposed (system level):

$$I_{rob,l} = \min\left\{\frac{\beta_{sys,dmg,l}}{\beta_{sys,\text{int}}};1\right\} \quad \forall\beta_{sys,dmg,l} \geq 0, \forall\beta_{sys,\text{int}} > 0 \qquad I_{rob} \in [0,1]$$

Ideal robust structure $I_{rob} = 1$

Non robust structure $I_{rob} = 0$

2. Sport centre in Samobor

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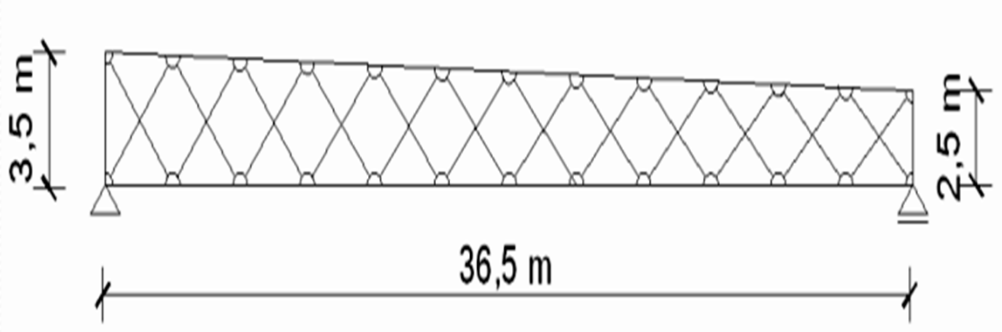
2. Sport centre in Samobor



The total area of the considered sport centre is 5910 m². It consists of three main parts: 1) main hall with dimensions 36,5x45 m, 9 (m) height for 600 visitors, 2) swimming pool with dimensions 12, 5x25, 10 (m) and depth from 1, 8 to 2, 4 (m) and 3) two smaller halls with dimensions 20x15 (m).
The structure was calculated according to Eurocode 5. The design was performed by Chair for the timber structures at the Faculty of Civil Engineering (prof. Rajcic), University of Zagreb

2. Sport centre in Samobor

For design characteristic values of permanent load (g= 6.38 kN/m), snow load (s=7.5 kN/m) and wind load (w=0.9 kN/m) are used. The material is timber GL32k. Based on the design the following cross section dimensions were chosen: **upper chord 20/52 cm**, **lower chord 20/69 cm and diagonal elements 20/24 cm.**

3. Probabilistic model

Reliability index is estimated by means of FORM

Second order effects are neglected for beams subjected to compression and combined compression and bending

Buckling problems and lateral buckling is taken into account as in Eurocode 5 with deterministic coefficients.

3. Probabilistic model

Identification of the significant failure modes of this structure is difficult to perform since there are many possible failure elements.

Assumed failure modes:

Failure in lower cord (N+M)

Failure due to tension in diagonal element (N)

Failure due to compression in diagonal element (N)

Failure in upper chord (N+M)

3. Probabilistic model

Limit state equations (for given failure elements)

$$g_1 = X - \frac{N_E}{0.8 \cdot ft \cdot b\_dp \cdot h\_dp \cdot k_{mod}} - 6 \cdot \frac{M_E}{0.8 \cdot fm \cdot b\_dp \cdot (h\_dp)^2 \cdot k_{mod}}$$

$$g_2 = X - \frac{N_E}{0.8 \cdot ft \cdot b\_d \cdot h\_d \cdot k_{mod}}$$

$$g_3 = X - \frac{N_E}{0.8 \cdot fc \cdot b\_d \cdot h\_d \cdot k_c \cdot k_{mod}}$$

$$g_4 = X - \frac{N_E}{k_c \cdot ft \cdot b\_dp \cdot h\_dp \cdot k_{mod}} - 6 \cdot \frac{M_E}{k_{crit} \cdot fm \cdot b\_gp \cdot (h\_gp)^2 \cdot k_{mod}}$$

3. Probabilistic model

Probabilistic variables

Label	Designation	Distribution	Mean value	COV
Es	MOE	LN	1170	0.13
X	Model uncertain.	LN	1	0.10
a	Joint distance	N	304.08	0.01
b_d	Width of diagonals	N	20	0.04
h_d	Height of diagonals	N	24	0.04
b_dp	Width - lower chord	N	20	0.04
h_dp	Height - lower chord	N	69	0.04
b_gp	Width - upper chord	N	20	0.04
h_gp	Height - upper chord	N	52	0.04
fc	Compression str.	L	2.66	0.12
fm	Bending str.	L	4.14	0.15
ft	Tension str.	L	2.48	0.18
g	Permanent load	N	0.068	0.10
s	Snow load	G	0.030	0.58

*Mean values of strengths are in kN/cm^2, dimensions in cm, actions in kN/cm^1.

4. Results

The element reliability indices

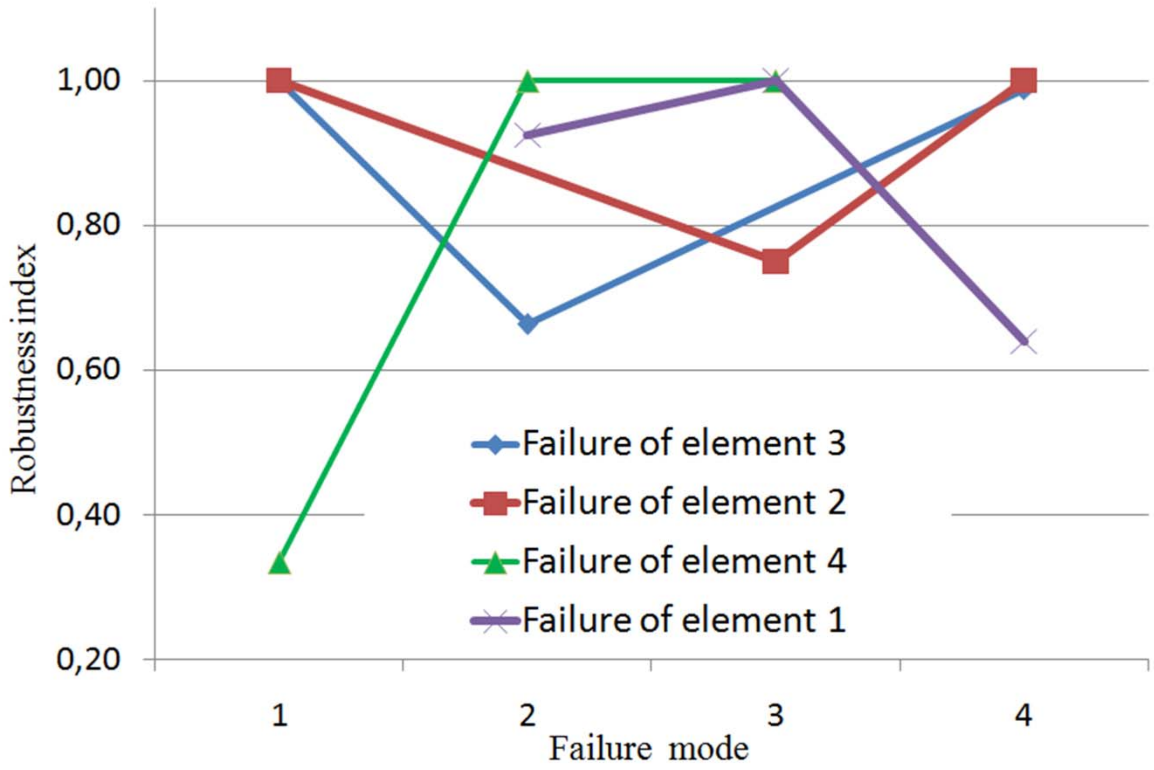
Element number	Beta index
1	4.99
2	7.76
3	7.04
4	4.46

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences failure
Large (A)	$\beta = 3.1\ (P_f \approx 10^{-3})$	$\beta = 3.3\ (P_f \approx 5 \cdot 10^{-4})$	$\beta = 3.7\ (P_f \approx 10^{-4})$
Normal (B)	$\beta = 3.7\ (P_f \approx 10^{-4})$	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$
Small (C)	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$	$\beta = 4.7\ (P_f \approx 10^{-6})$

Target reliability values for ultimate limit states (JCSS 2001)

4. Results

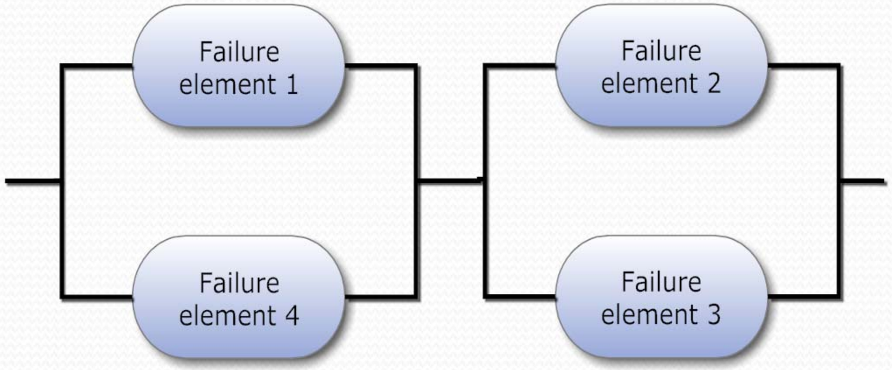
ANALYSIS BASED ON COMPONENTS



For an assumed failure of element 4 (e.g. failure in the middle of upper chord) the reliability index for element 1 is decreased to approx. 30% of intact structure
No extensive failure of the structure when elements 2, 3 and 4 fail

4. Results

ROBUSTNESS ON SYSTEM LEVEL



System model of the structure

The FORM approximation of a parallel system:

$$p_f = P\left(\bigcap_{i=1}^{n}\{M_i \le 0\}\right) = P\left(\bigcap_{i=1}^{n}\{g_i(X) \le 0\}\right)$$

$$p_f \approx P\left(\bigcap_{i=1}^{n}\left\{\beta_i^J - \alpha_i^T \cdot U \le 0\right\}\right)$$

$$= \Phi_{n,A}(-\beta^j, \rho)$$

$\Phi_{n,A}$: multivariate n-dimensional Normal distribution function

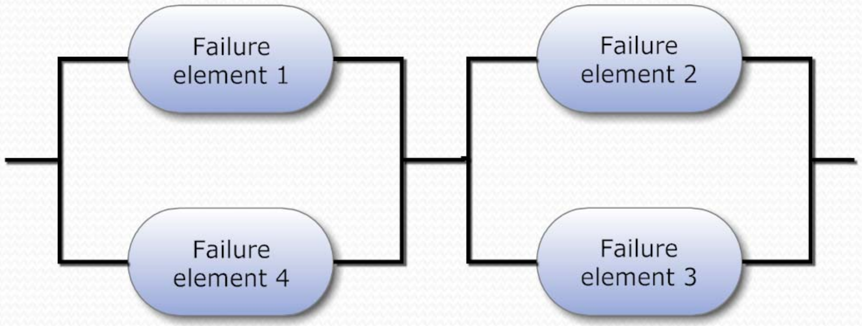
ρ: correlation matrix

4. Results

EVALUATION OF THE SYSTEM RELIABILITY

An estimate of the failure probability is: obtained as the arithmetic mean of the upper and lower probability bounds:

$$P_f^S \approx \frac{\max\left[p_{f,i}^P\right]_{i=1}^{j} + 1 - \prod_{i=1}^{j}(1 - p_{f,i}^P)}{2}$$



$\beta_{par} = 5.33$

$P_f = 5\times10^{-8}$

$P_f = 2.23\times10^{-16}$

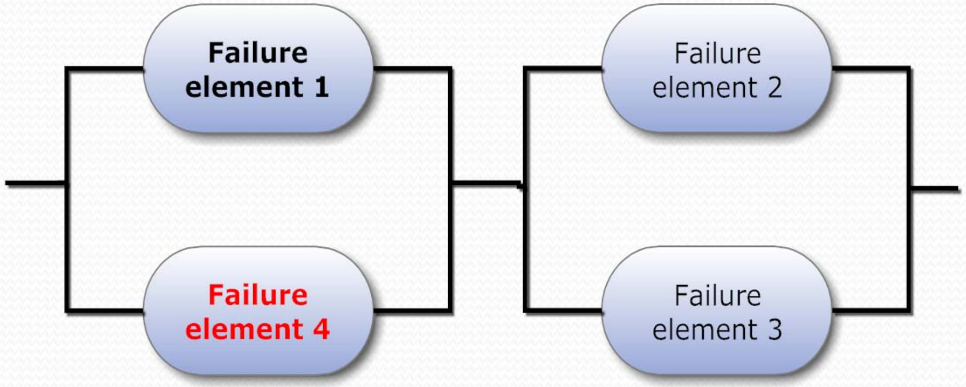
System reliability

$\beta_{sys} = 5.33$

$P_f = 5\times10^{-8}$

4. Results

For every component a failure is assumed and a system reliability is calculated :



$\beta_{par} = 1.67$

$P_f = 4.79 \times 10^{-2}$

$\beta_{par} = 8.12$

$P_f = 2.23 \times 10^{-16}$

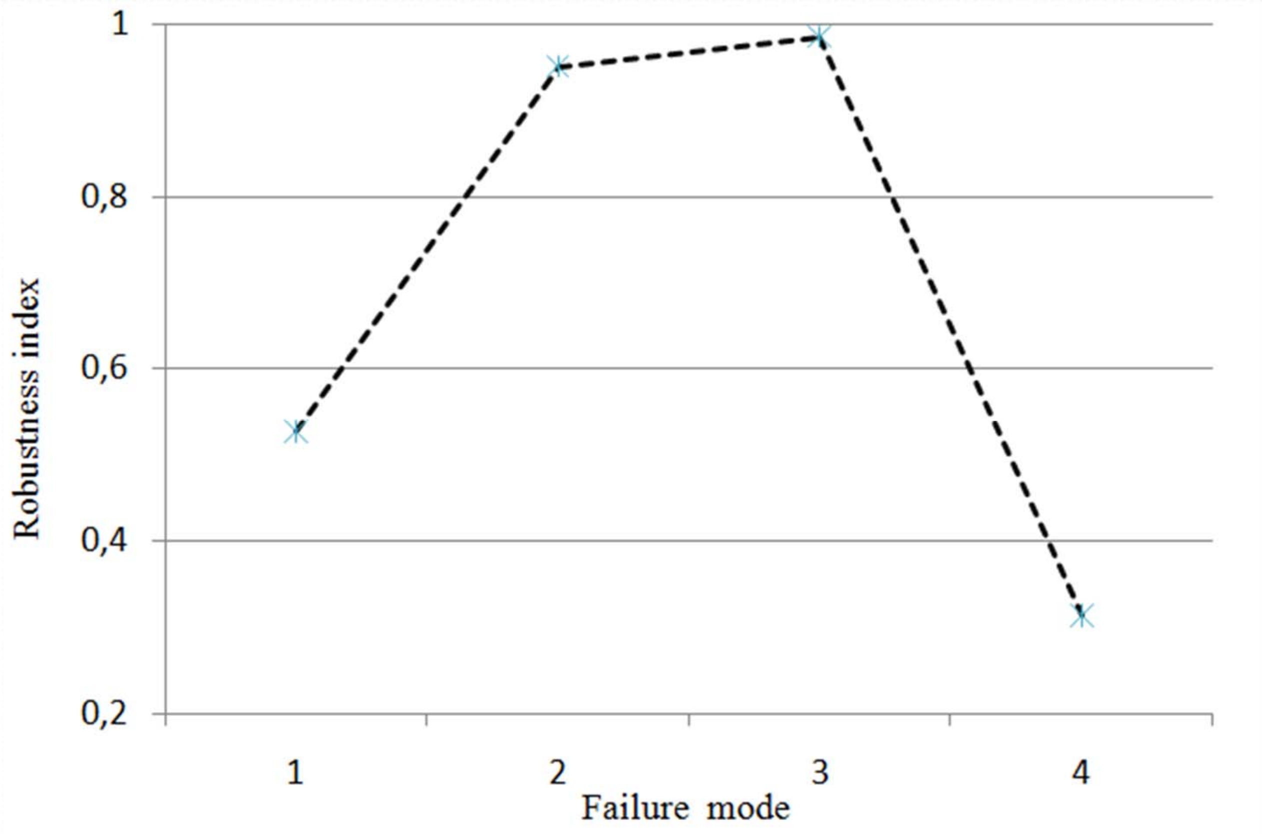
System reliability

$\beta_{sys} = 1.67$

$P_f = 4.79 \times 10^{-2}$

Example of failure of element 4

4. Results



- It can seen that the lowest system reliability occurs when element 4 is in failure.
- For the assumed damages in the elements 2 and 3 (e.g. tensile and compressive elements) no significant effect on the system reliability is observed, so the robustness index is high

5. CONCLUSION

FURTHER RESEARCH

- Preliminary research of ductility influence (in compression) shows higher robustness indices of components - it is expected the same on system level
- Investigation on ductility of connections is also underway

THANKS FOR ATTENTION

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