Evolution of Probabilistic Analysis of Timber Structures from Second-Moment Reliability Methods to Fragility Analysis

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- Trend worldwide toward probabilistic structural design
- Reliability-based design standards for timber (wood) evolved in the US and Canada in the 1980's and 1990's
- 20^{+} years later, where are we?
  - What did we accomplish? How did we get there?
  - What worked? What hasn't?
  - What has evolved? How? Why?
  - Where are we going?

Introduction

US. Load and Resistance Factor Design (LRFD) (1)

$$\phi R_n \geq \sum_i \gamma_i Q_{n_i}$$

National Building Code of Canada (2)

$$\phi R_n \geq \gamma_D D_n + \psi\left(\sum_i \gamma_i Q_{n_i}\right)$$

General Partial Safety Factor Design (Eurocode) (3)

$$\frac{R_n}{\gamma_{r_1}\gamma_{r_2}\gamma_{r_3}} \geq \gamma_{s_1}\gamma_{s_2}\gamma_{s_3}\left[fcn\left(\sum_i Q_{n_i}\right)\right]$$

Structural reliability can be evaluated by computing the probability that a particular limit state function is less than zero. The function is expressed for a particular limit state and a particular load combination. The failure probability can be expressed,

$$P_f = P[g(x_1, x_2, \ldots, x_n)] < 0 \quad (4)$$

Reliability-based design

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$$P_f = P[g(x_1, x_2, \ldots, x_n)] < 0 \quad (4)$$

- All relevant **limit states** considered (flexure, shear, deflection, etc.) in design process
- All relevant **load combinations** checked to determine controlling combination(s)
- Load combination rules and **partial safety factors** taken from (e.g.) ASCE 7

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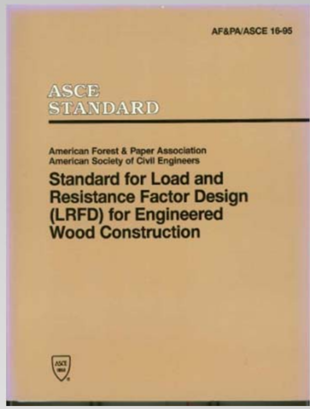
FOSM ► FORM / SORM / AFOSM ► MCS ► AMCS

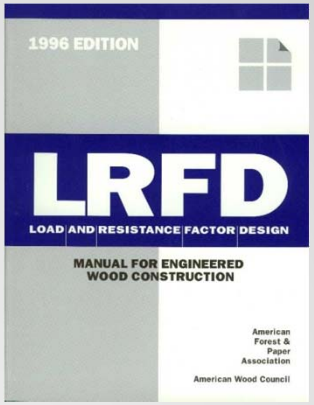
- Murphy et al., 1988 (US)
- Foschi et al., 1989 (Canada)
- ASCE 7 (load factors, load combinations), AISC and ACI
- In-Grade Test Program (1987, 8 vols.) 
- Soft calibration to NDS (ASD), design strength values, target reliabilities
- Time-effects factors (cumulative damage models for species groups)
- Connections
- Repetitive member system factors

Result: LRFD for Wood

- **ASCE 16-95 Standard** released in 1996



- **LRFD Manual** (AF&PA) released in 1997



Changes in Probabilistic Modeling Approaches, Risk Analysis

- ***Fully coupled*** analysis
  - Loads and resistances treated explicitly, simultaneously
  - Reliability-based code calibration
  - "R-S" analyses, FORM-SORM
- ***Uncoupled*** risk analysis
  - Dominant source of uncertainty (e.g., extreme load)
  - Separates response from the hazard
  - Fragility analysis
- ***Partially coupled*** analysis …

- ***Partially coupled*** analysis
  - Characteristic suite of (e.g., scaled ground motions) selected to characterize the hazard
  - Probabilistic response description (e.g., CDF) developed, median-based mechanical and structural properties
  - Response distributions and performance requirements (e.g., drift limits) then form the basis for design tables/charts

Changes in Probabilistic Modeling Approaches, Risk Analysis

Fully coupled analysis – classical reliability analysis (e.g., FORM)

$P_f = P[g(\mathbf{x}) < 0] = P[g(\theta_{load}, \theta_{resistance}, \theta_{system} ; t) < 0]$ (5)

LRFD, member-based

$\beta = \Phi^{-1}(1 - P_f)$; β = reliability index

$\Phi^{-1}(\,)$ = inverse standard normal cumulative distribution function (CDF)

Uncoupled analysis – fragility (conditional probability) analysis

Assessment (assembly or system level)

$Fr(x) = P[LS \mid D]$; $Fr(x)$ = fragility, LS = limit state, D = demand (6)

$P_f = P[LS] = \int P[LS \mid D]\, P[D]$; P[D] given by hazard function

Partially coupled analysis – probabilistic response (performance) analysis

$\Lambda(\rho) = \Lambda(\Omega_{hazard}, \theta_{resistance}, \theta_{system} ; t ; \rho)$ = response quantity (7)

Assembly selection (design)

ρ = design parameter, Ω = suite of records (*e.g.*)

$F_\Lambda(x)$ = cumulative distribution function (CDF) of response quantity Λ

Probabilistic risk analysis for design

closed-form LS function

AFOSM can be used

Easier than DOL/MCS ($\Sigma\alpha_i$) or order-statistics

$$g(\mathbf{x}) = A - B\left[\frac{\prod c_i \lambda \phi}{R / R_n}\left(\frac{1}{\gamma_D + \mu\gamma_X}\right)\left(\frac{D}{D_n} + \mu\frac{X}{X_n}\right)\right] - \ln \Delta t$$

product of adjustment factors

nominal load ratio

Range of values determined for various species/size/grades (dimension lumber)

EDRM damage model parameters

load factors

variable load

critical pulse duration

can be changed

ASD or LRFD

"killer pulse" concept

Single member limit state function (gravity loads)

B/F-II (Forintek) Model

$$g(\mathbf{x}) = 1 - \ln\left(\frac{A}{C}\right) - B\left(\ln\left[\frac{\Pi\, c_i \lambda \phi}{R/R_n}\left(\frac{1}{\gamma_D + \mu\gamma_X}\right)\left(\frac{D}{D_n} + \mu\frac{X}{X_n}\right)\right] - \sigma_0\right) - C\tau$$

product of adjustment factors

nominal load ratio

load factors

critical pulse duration

B/F-II damage model parameters

variable load

Single member limit state function (gravity loads)

$g(\underline{x};t) = g[\underline{x}_{load}(t), \underline{x}_{resistance}(t), \underline{x}_{system}(t)]$ *limit state function*

$P_f = \Pr(\alpha(t) > 1;\ 0<t<T_{ref})$ ***cumulative damage***

- **Updated load process models, statistics**
- **Resistance statistics based on IGTP data, species groupings**
- **Comparison of cumulative damage models for similar species, validation**
- **Load duration (time effects) factors**
- **Interaction effects:**
  - **Repetitive-member systems**
  - **Moisture content**
  - **Beam-columns**
  - **Roof ponding**

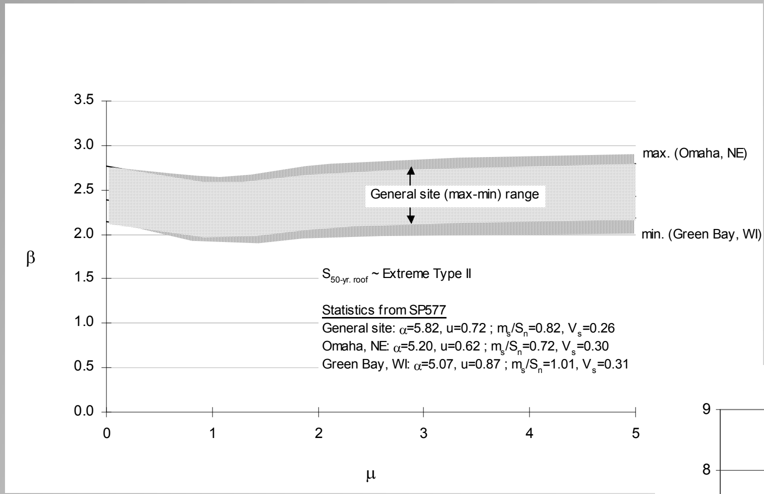
Single-member limit state analysis with cumulative damage (time-dependent simulation)

$\beta_{FOSM}(x)$

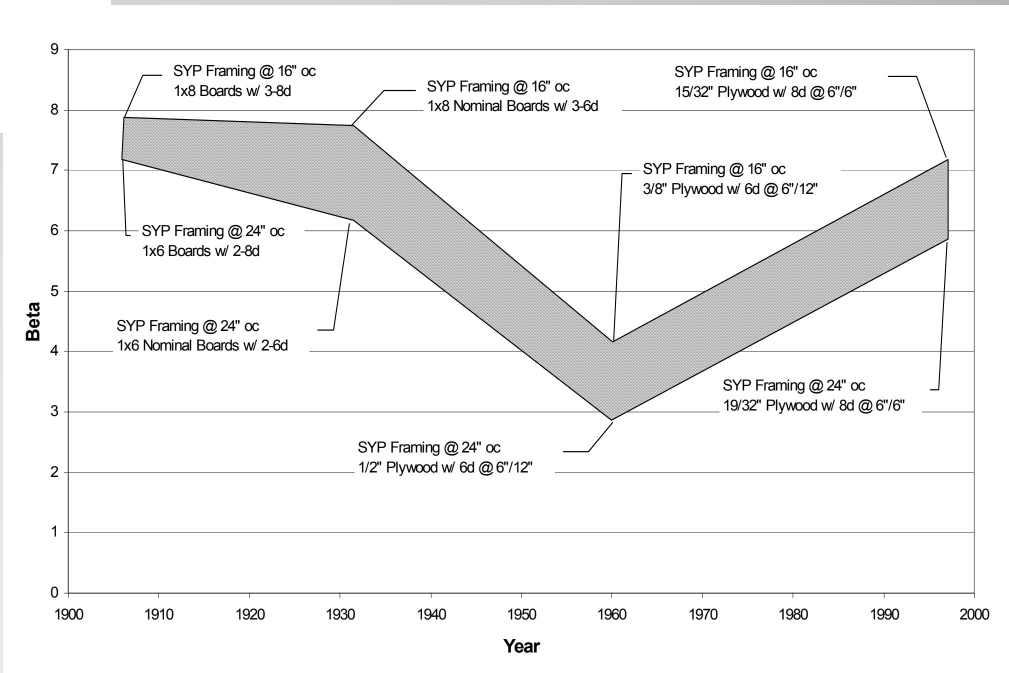
- Partial factors (ϕ)
- DOL factors (λ)
- System factors (ψ_{sys})
- MC/exposure factor (θ)
- …

Results

$\beta_{FOSM}(x)$



β
μ
max. (Omaha, NE)
min. (Green Bay, WI)
General site (max-min) range
S50-yr. roof ~ Extreme Type II
Statistics from SP577
General site: α=5.82, u=0.72 ; ms/Sn=0.82, Vs=0.26
Omaha, NE: α=5.20, u=0.62 ; ms/Sn=0.72, Vs=0.30
Green Bay, WI: α=5.07, u=0.87 ; ms/Sn=1.01, Vs=0.31



Beta
Year
SYP Framing @ 16" oc
1x8 Boards w/ 3-8d
SYP Framing @ 16" oc
1x8 Nominal Boards w/ 3-6d
SYP Framing @ 16" oc
15/32" Plywood w/ 8d @ 6"/6"
SYP Framing @ 16" oc
3/8" Plywood w/ 6d @ 6"/12"
SYP Framing @ 24" oc
1x6 Boards w/ 2-8d
SYP Framing @ 24" oc
1x6 Nominal Boards w/ 2-6d
SYP Framing @ 24" oc
19/32" Plywood w/ 8d @ 6"/6"
SYP Framing @ 24" oc
1/2" Plywood w/ 6d @ 6"/12"

IGTP → Assembly tests → CUREE → NEESWood

IGTP
- Coupled analyses
- Code calibration
- RBD
- Time effects
- System factors

Assembly tests
- Shear walls
- Diaphragms
- Walls
- Connections

CUREE
- Partially coupled analyses
- 3D structural modeling
- Portfolio analyses
- Hazard characterization (suite of scaled records)
- Shear wall selection
- Design charts

NEESWood
- Uncoupled analyses
- Fragilities
- PBSD
- DDD

A transition (spanning ~25 years)

IGTP → Assembly tests → CUREE → NEESWood

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System Factor Definitions

- Geometric (section properties), e.g., ratio of PCM to bare stud section modulus
- Strength-based, e.g., ratio of system to individual member ultimate strength
- Reliability-based, e.g., bring system reliability down to member reliability (assumes comparable failure consequences)
- Others (e.g., ratio of ultimate-to-yield, etc.)

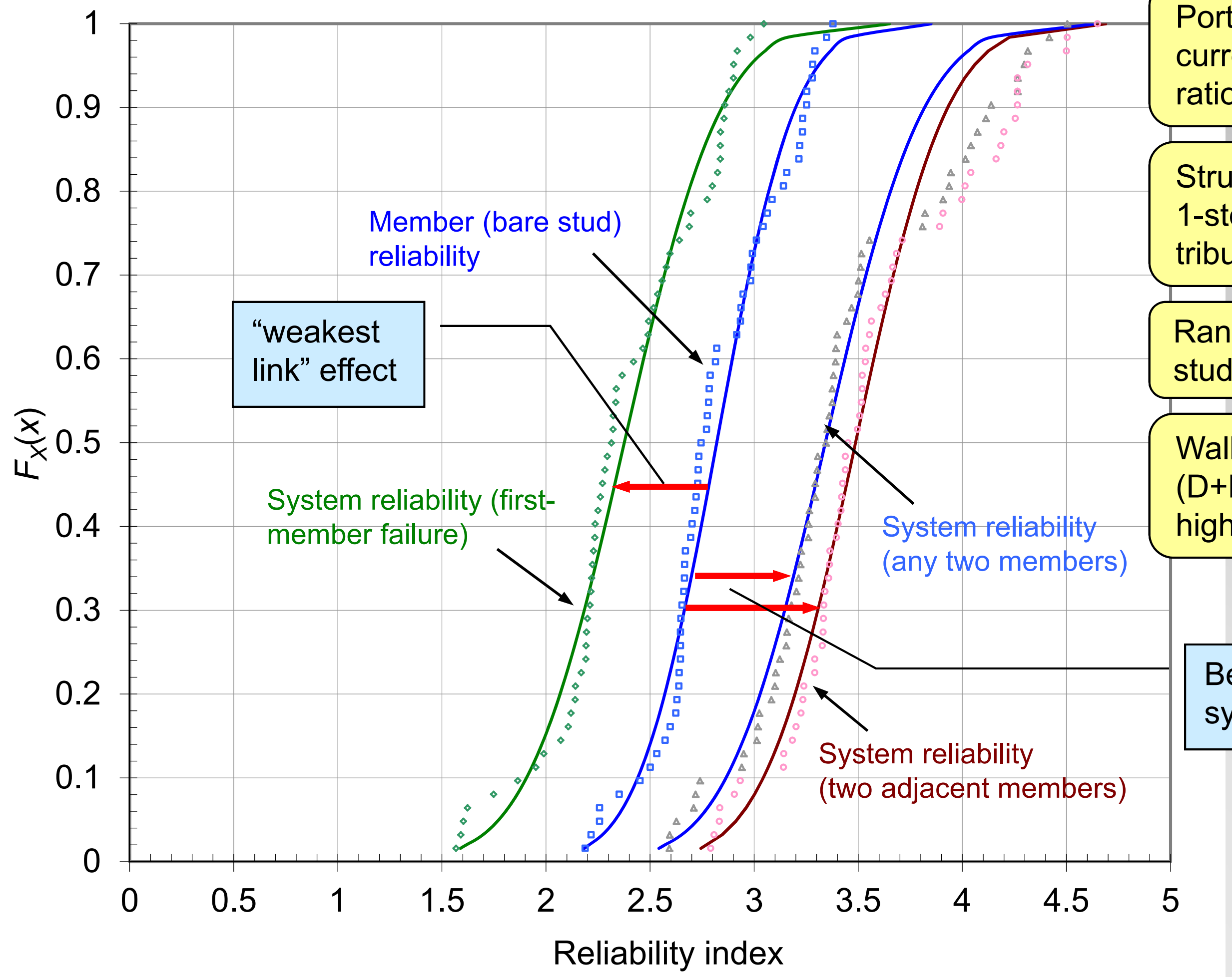
Repetitive member factors

Portfolio approach

- In some cases, it may not be possible to express a generalized limit state function in terms of nominal values
  - e.g., indeterminate systems in which complex material behavior, load-sharing behavior, and/or system limit state definitions are being considered

- An alternative to using a generalized limit state function with a bounded basic variable set is to consider a "portfolio approach" 
  - range of explicit systems
  - assumed to be representative of the design space

Repetitive-member system reliabilities



- Portfolio of walls designed to meet current code, with combined stress ratios close to 1.0
- Structures ranging in height from 1-story to 3-stories, with typical tributary floor/roof areas.
- Range of stud species/size/grades, stud spacings, wall heights, etc.
- Walls subject to combined axial (D+L+S) and transverse (W) loads; high wind region.

Example: wood stud walls

- Single factor (e.g., on flexural strength) may not be adequate for all system configurations, materials, and load types.
- Evaluation of system factors for design of wall members
  - ✓ Compatible with current format (e.g., C_r=1.15) in NDS and LRFD
  - ✓ Proposed new format for repetitive-member factors:

$$K_{sys} = \Pi K_{PCA} K_{NMEM} K_{PY} K_{LS}$$

Partial system factors

Partial system factors

$$K_{sys} = \Pi \, K_{PCA} \, K_{NMEM} \, K_{PY} \, K_{LS} \, K_{calib.}$$

K_{PCA} — Partial composite action

Ratio of PC member to bare member strength

K_{NMEM} — System size (# members)

Ratio of series system to PC member strength

K_{PY} — Member post-yield behavior (redundancy)

Ratio of system strength defined by first-member failure to series system

K_{LS} — System load-sharing behavior

Ratio of system strength defined by system ultimate to first-member

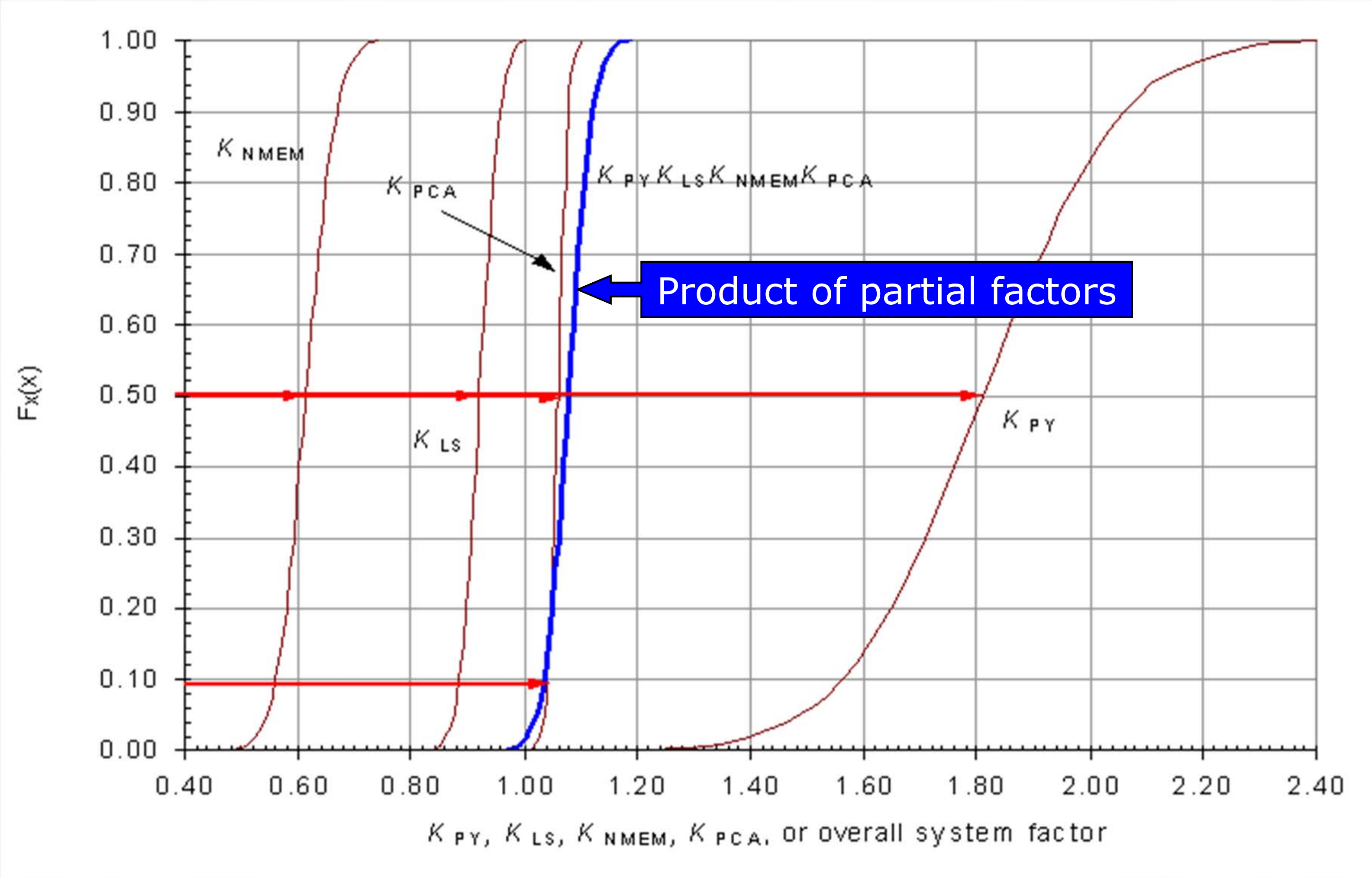
$K_{calib.}$ — Calibration factor

Professional factor

Ratios of 5th-percentiles?

Mixed ratios? (medians, 5th-percentiles)

Some ratios deterministic?



Example: wall with openings on both sides

IGTP → Assembly tests → CUREE → NEESWood

IGTP
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- System factors

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A transition (spanning ~25 years)

(A ^sort of new paradigm)

Performance-based engineering

- Design process is structured to meet specific performance expectations of the building occupants, owner and public
- Gaining momentum in North America, Japan, and elsewhere
- First discussed in 1970's (HUD "Operation Breakthrough")
- Revisited in 1990's, following Loma Prieta and Northridge, when it became apparent that buildings design by code for life safety often did not meet performance expectations in other aspects ($$$)
  - SAC Steel (MRF) project
  - CUREE-Caltech Wood Frame project

Background: Structural reliability, Single-member checking equations

- Structural reliability theory has been used as the basis for code development since the 1970's
- LRFD for wood, performance requirement (safety):

$$\lambda \phi R' > \Sigma \gamma_i Q_i$$

- ***Single-member*** checking equations (members, components, connections) used in design of new structures

Shortcomings:

- Provide only an approximate picture of how a ***system*** of such members performs
- Unable to provide meaningful information on expected performance of a large number of (existing) structures

PBE concepts

- PB framework typically based on 3-4 generally stated **performance goals**, e.g.,
  1. IMMEDIATE OCCUPANCY following moderate events (local or no damage)
  2. LIFE SAFETY under design-basis events (moderate damage)
  3. COLLAPSE PREVENTION under maximum considered events
- Challenge: state goals must be expressed in terms of structural responses that the engineer can evaluate with available analytical tools.

PBE concepts (cont'd.)

System reliability

Analytical models of system performance

- Complete systems (e.g., building frames)
- Sub-systems, assemblies (e.g., shearwalls)

Nonlinear FE models	Time-history analysis
Multiple failure modes	Integrated failure modes

Fragility modeling

Uncoupled (vs. fully coupled) risk analysis

Uncouples the system analysis from the hazard

- Aleatory (variability) vs. epistemic uncertainty
- Unlike **fully coupled** approach (e.g., FORM) taken in developing limit states design, one source of variability often dominates ($V_S >> V_R$)
- An **uncoupled** fragility analysis provides a useful framework (e.g., for assessment) and suggests alternate approaches

Design for natural hazards, fragility analysis

Fragility of structural system often modeled by a Lognormal CDF:

DEMAND

MEDIAN CAPACITY

$$Fr(x) = \Phi\left[\frac{\ln(x/m_R)}{\xi_R}\right]$$

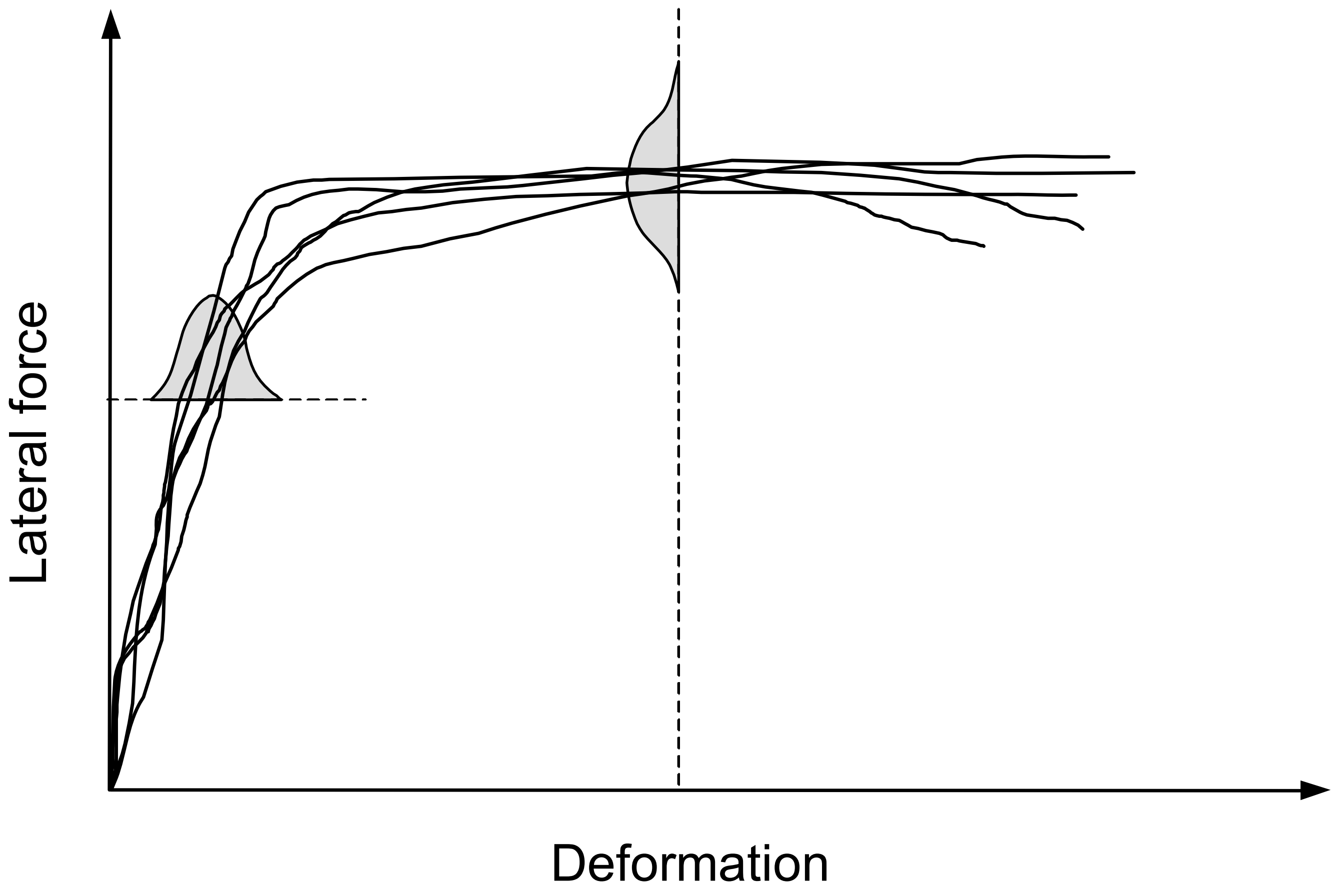
≈ C.O.V. OF CAPACITY

- Possible first-order estimation of m_R by single nonlinear analysis, or small n
- ξ_R relatively insensitive to small variations in design parameters for one class of structural systems

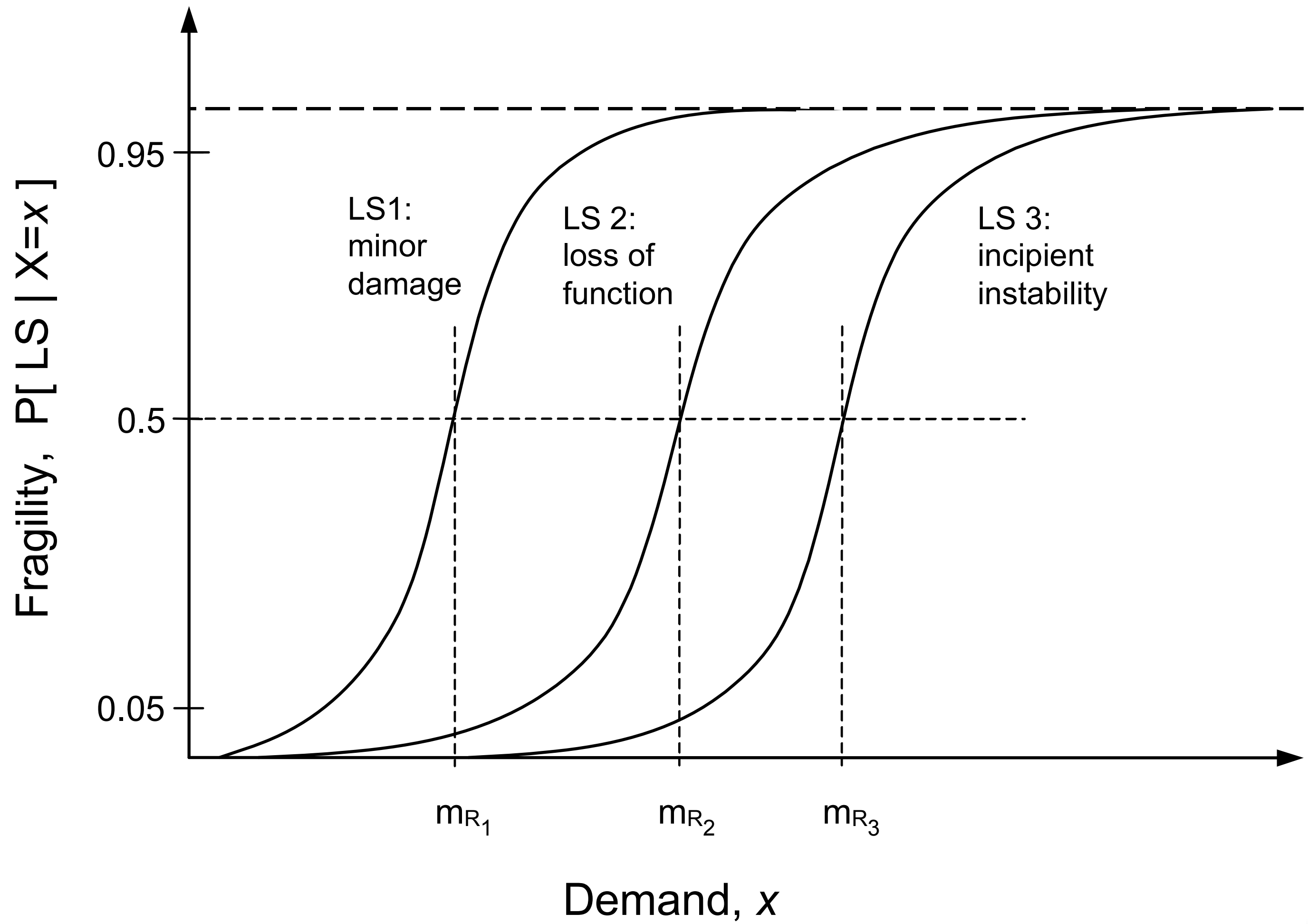
Fragility modeling

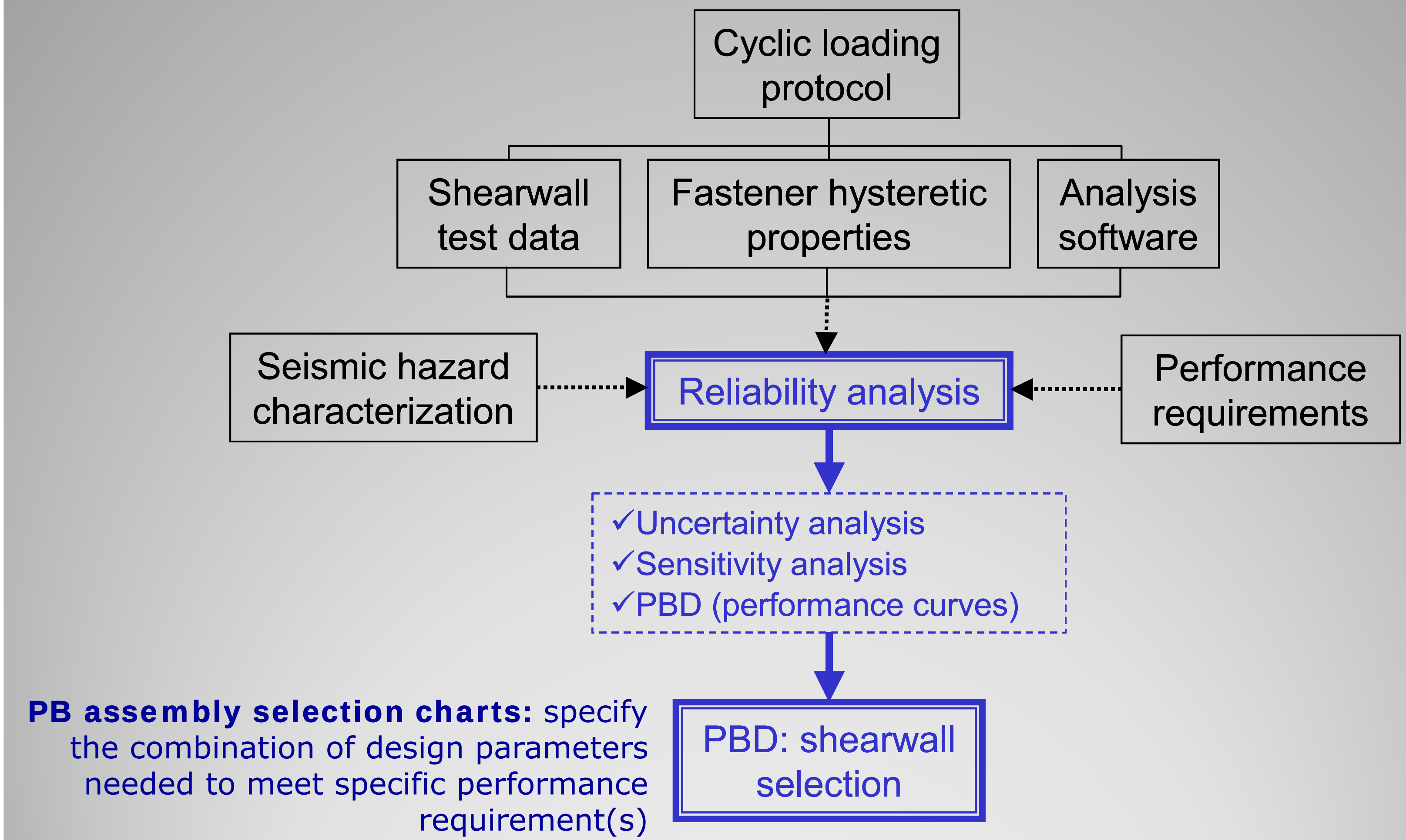
Load-deformation curves

(e.g., from a nonlinear finite element analysis)



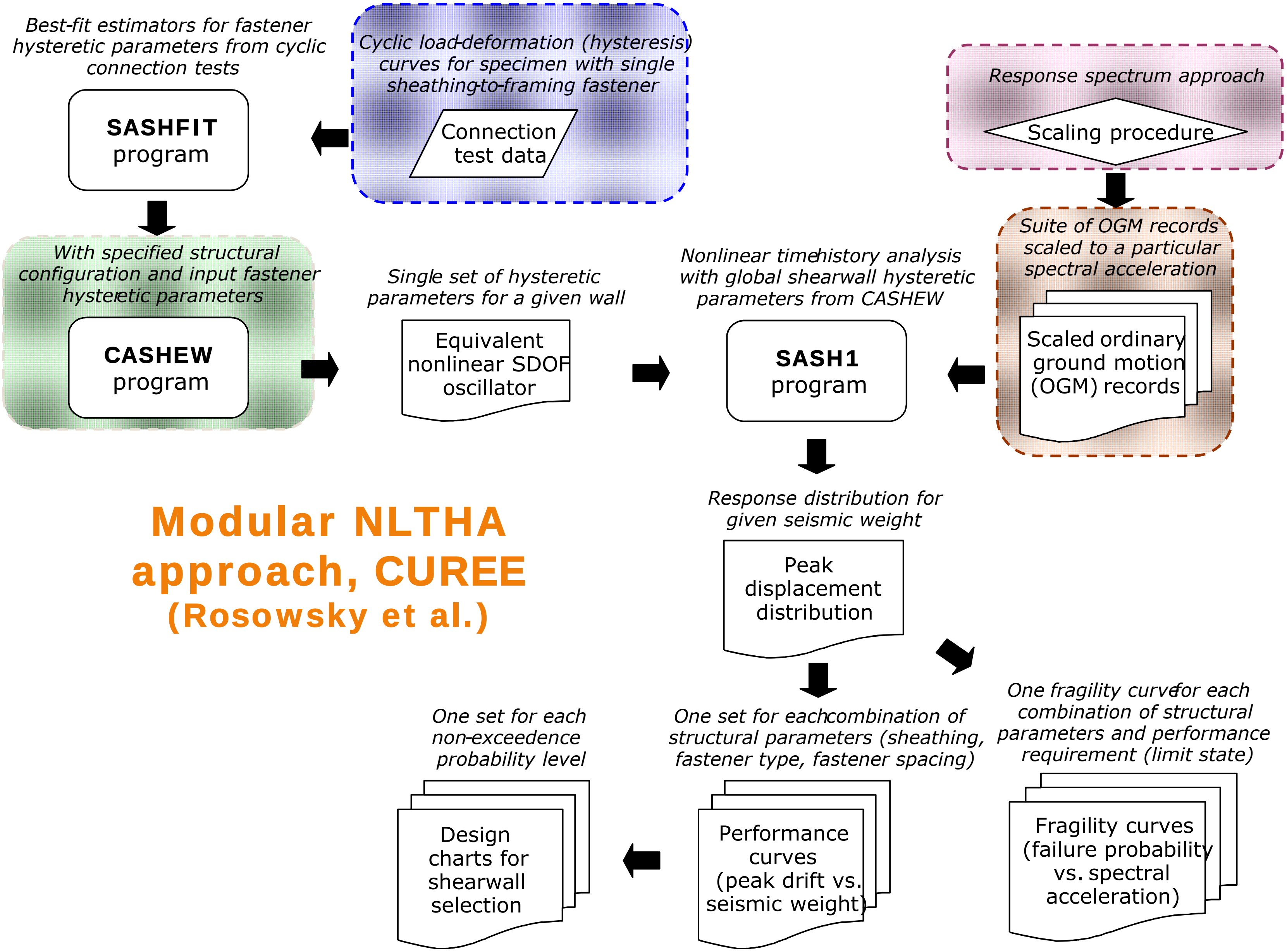
Suite of fragility curves





CUREE-Caltech Woodframe Project

Modular NLTHA approach, CUREE (Rosowsky et al.)



Performance-based (seismic) design

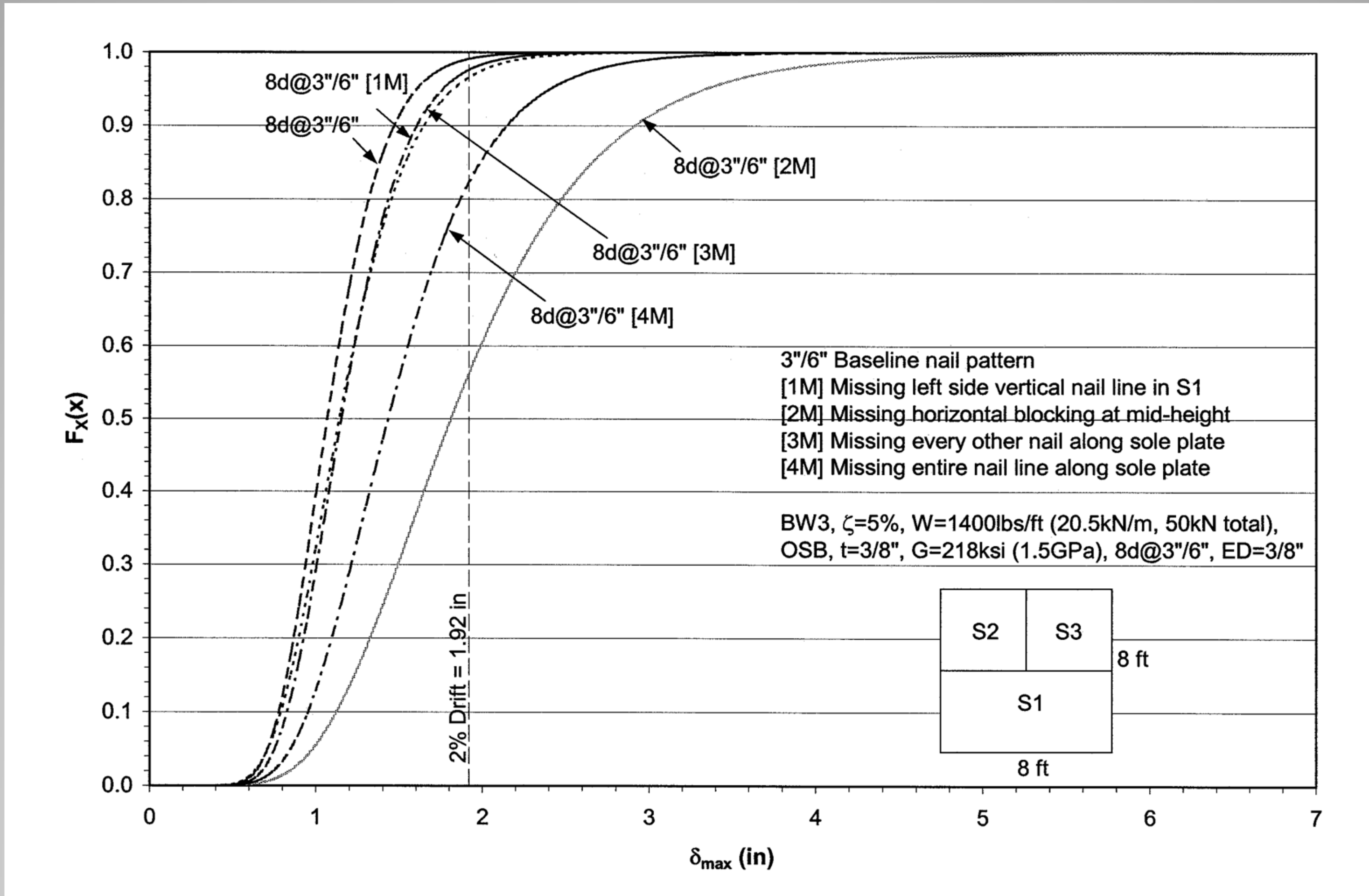
- Peak displacement distributions
  - Assembly-level
  - Full structure
- Performance curves, design charts
- Fragilities, $Fr(x)$
- Direct Displacement Design (DDD)
- Performance-based DDD

- Peak displacement CDF's can be post-processed into a form more useful for design (dependent variable: **seismic weight**)
- **Performance curves** are intermediate step toward developing **design charts**
- Peak displacement CDF's (non-parametric) can be post-processed into **fragility curves,** $Fr(x)$ = P[LS|D]

Post-processing results

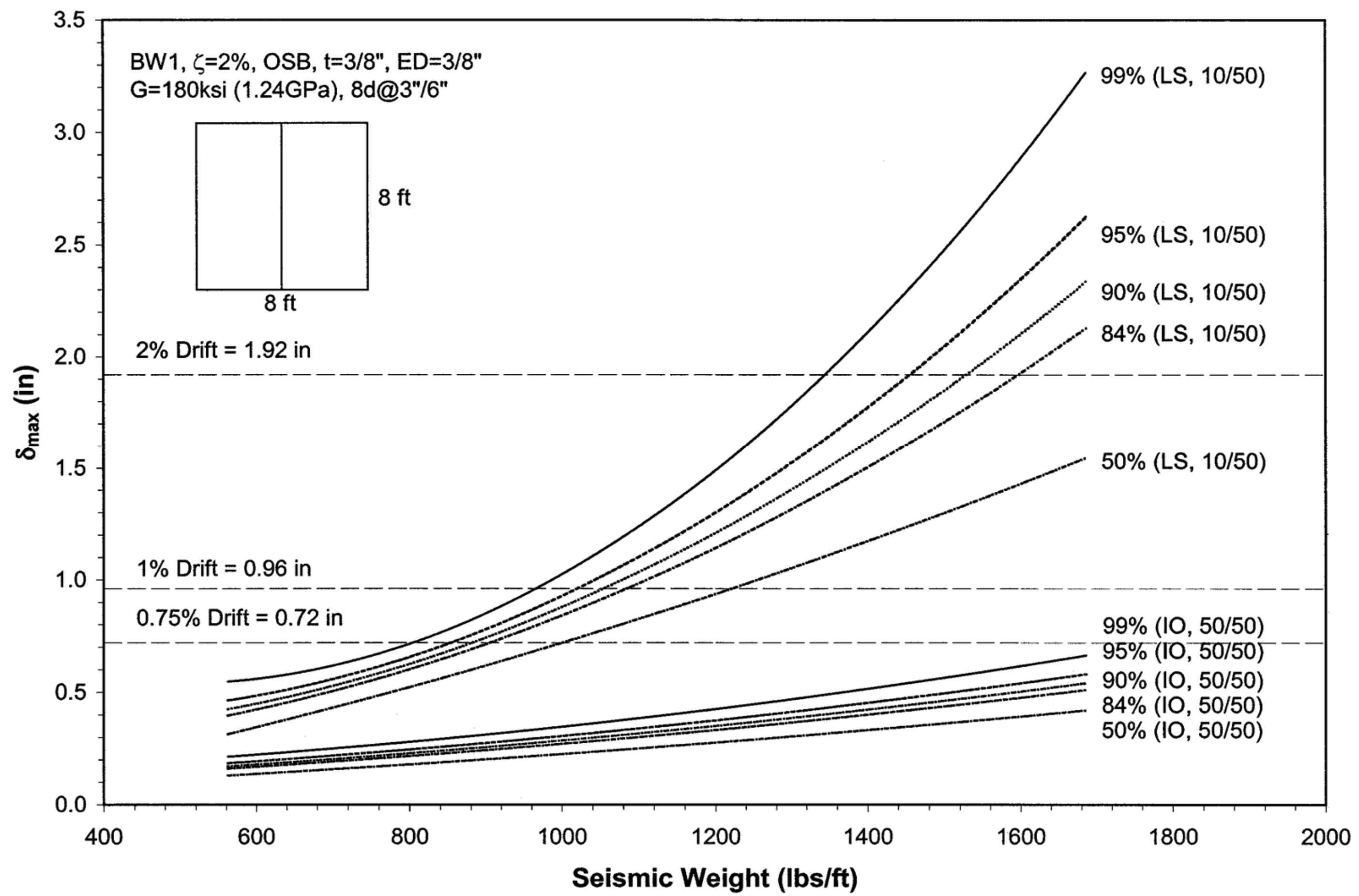
Evolution of PBSD for Wood Structures

- Simplified design charts ✓
- Fragilities for assessment ✓
- Fragilities for design x
- PBSD (DDD) ✓

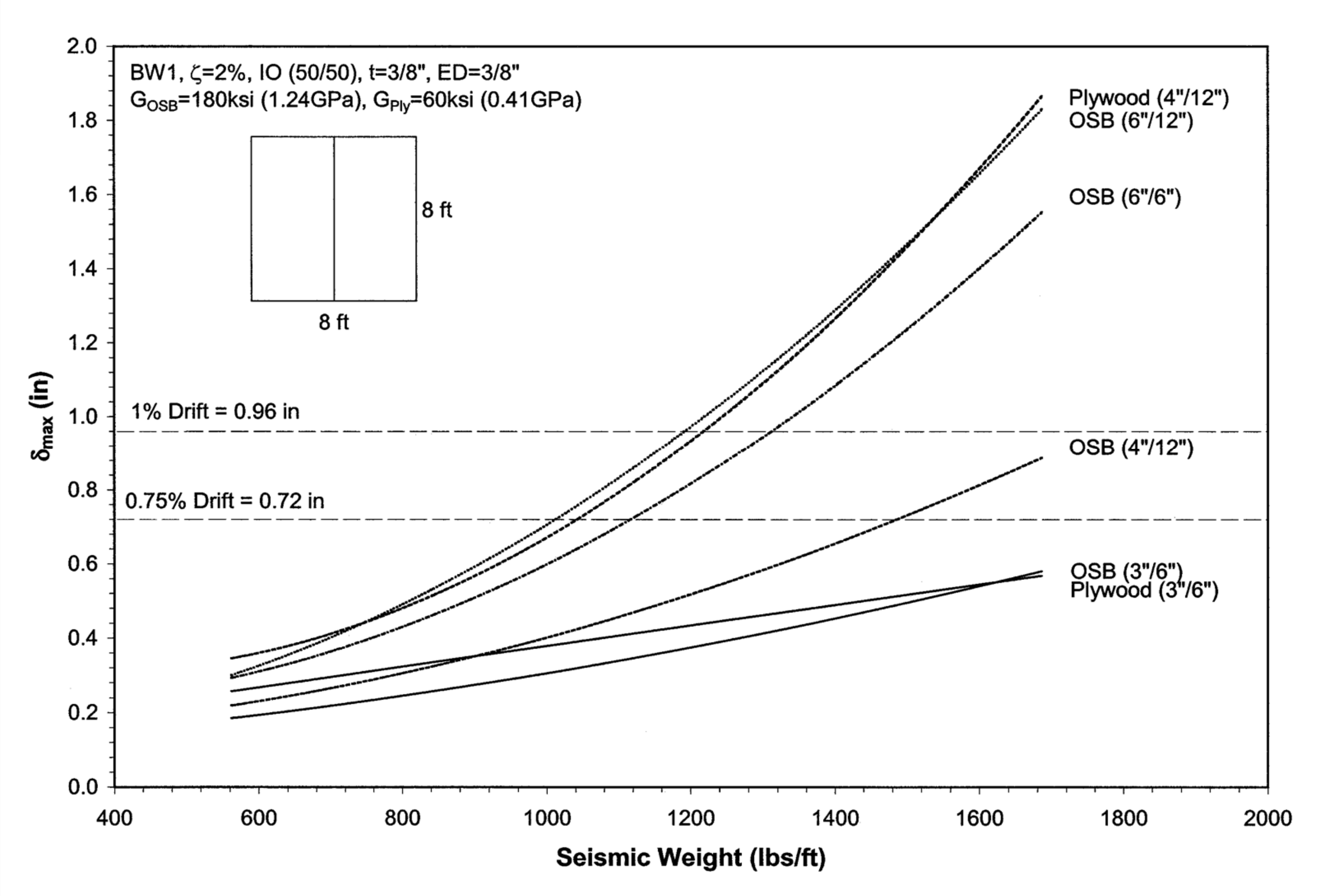


Assembly-level peak displacement distributions

(effect of missing fasteners)



Assembly-level performance curves



Assembly-level design chart

Fragilities

Peak displacement CDF's (non-parametric) can be post-processed into (parametric) **fragility curves**, $\mathrm{Fr}(x)$ = P[LS|D]

Performance-based **Assessment** (PBA)

Performance-based **Design** (PBD)

"Performance-based Engineering (PBE)"

Fragility equation

$$F_R(C \le D \mid S_a) = 1 - \Phi\left(\frac{\lambda_C - \lambda_D}{\sqrt{\beta_C{}^2 + \beta_D{}^2 + \beta_M{}^2}}\right)$$

Log Median Capacity

Log Median Demand

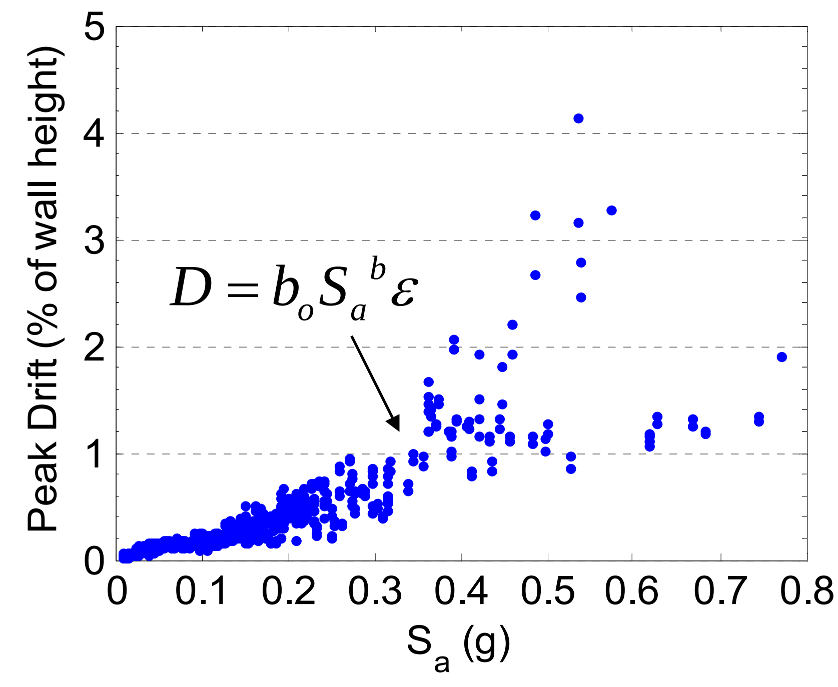
Capacity Uncertainty

Demand Uncertainty

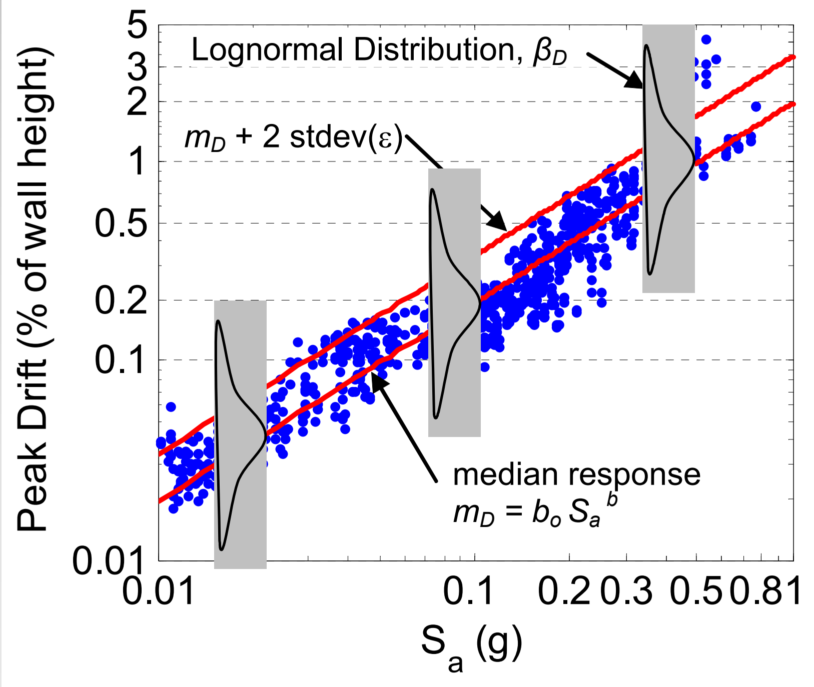
Modeling Uncertainty

aleatoric (inherent) and *epistemic* (knowledge-based) uncertainties are taken into account through the β terms

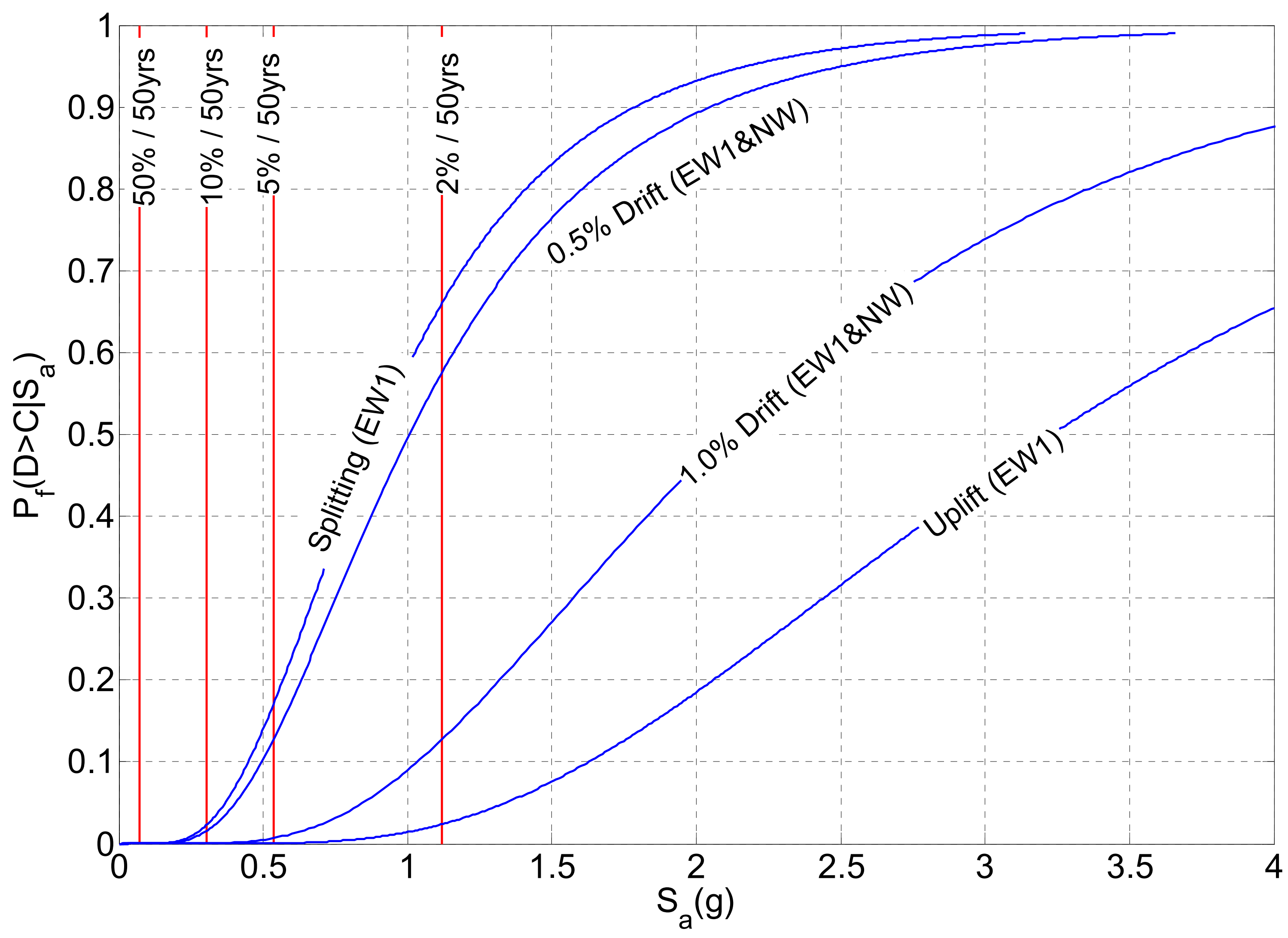
Original space



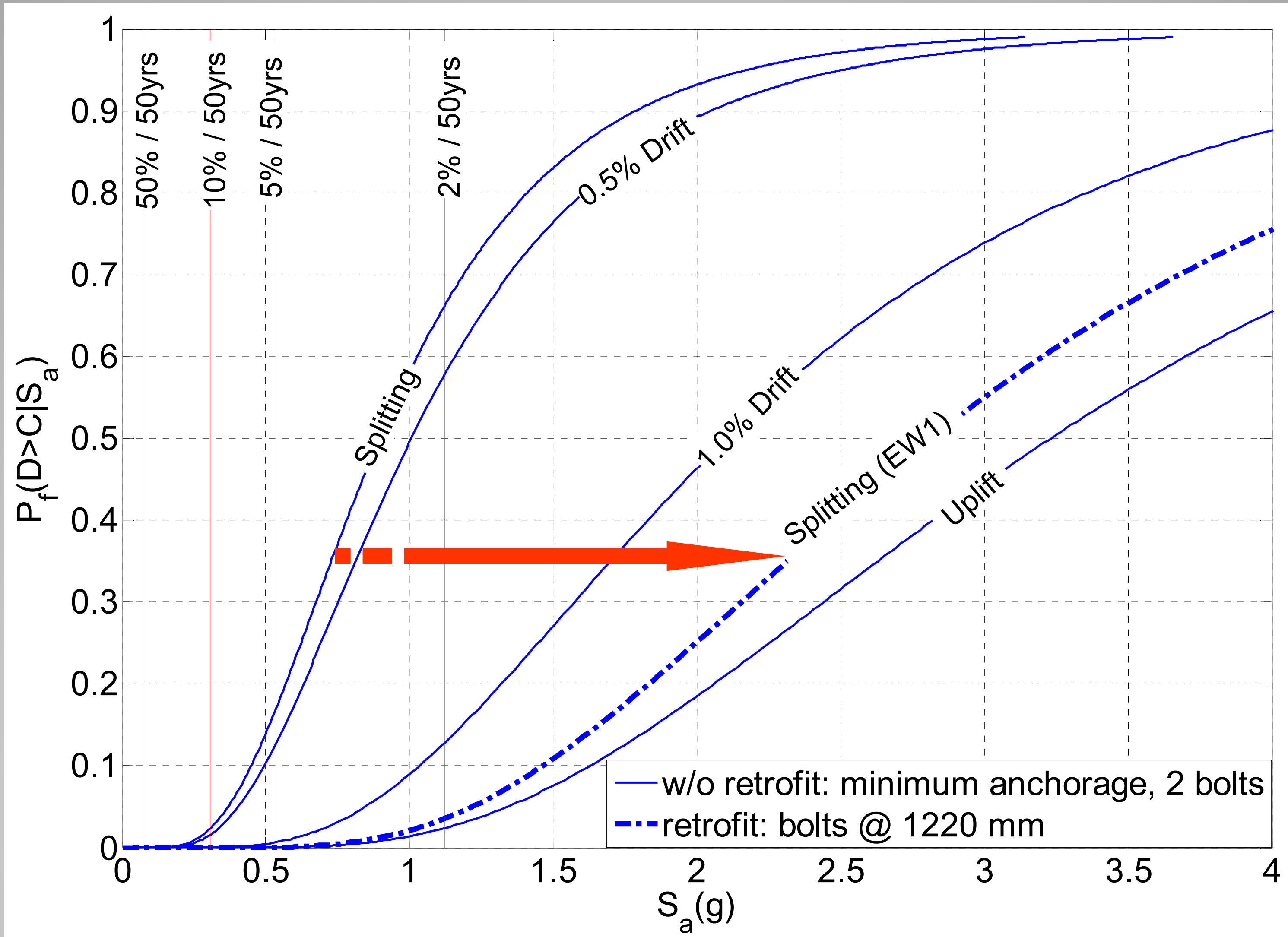
Log-log space



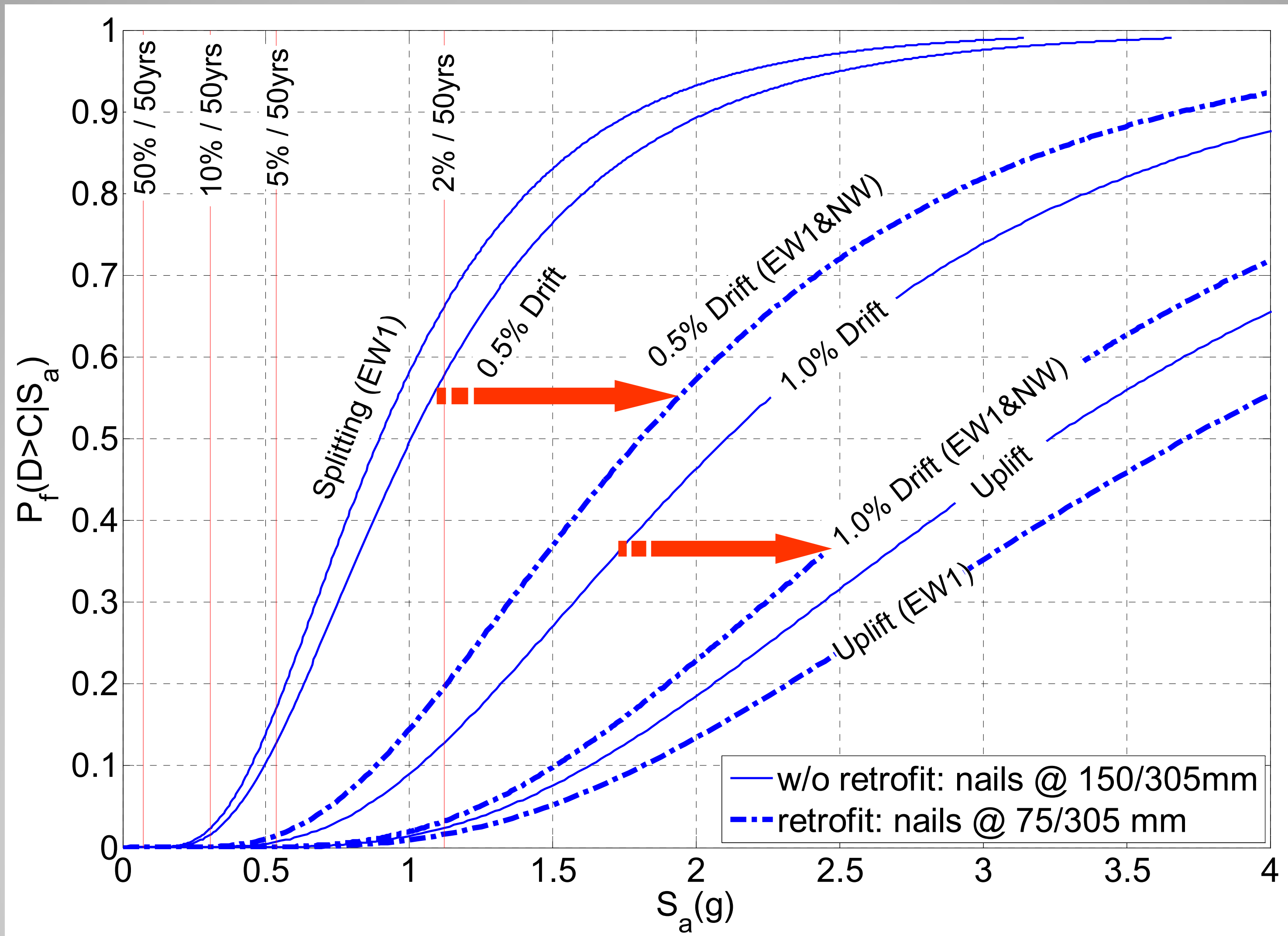
Determination of demand uncertainty



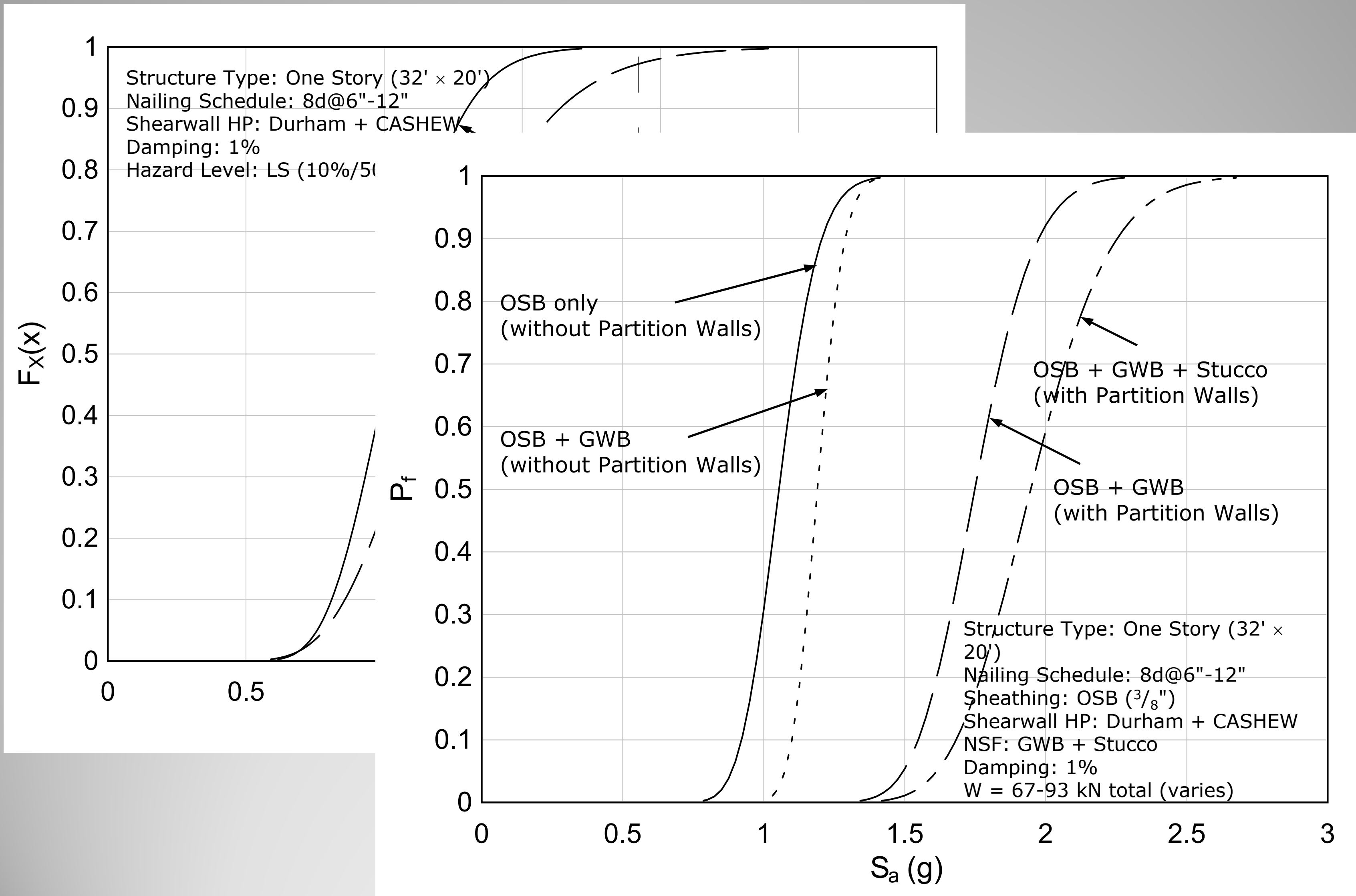
Ex., Fragility curves for one-story structure, isolated wall (3 modes)



Ex., One-story structure, isolated wall: retrofit evaluation (1)



Ex., One-story structure, isolated wall: retrofit evaluation (2)



Assembly *vs.* complete structure

Whole structure modeling and analysis
(NLTHA, seismic response characterization, PBSD)

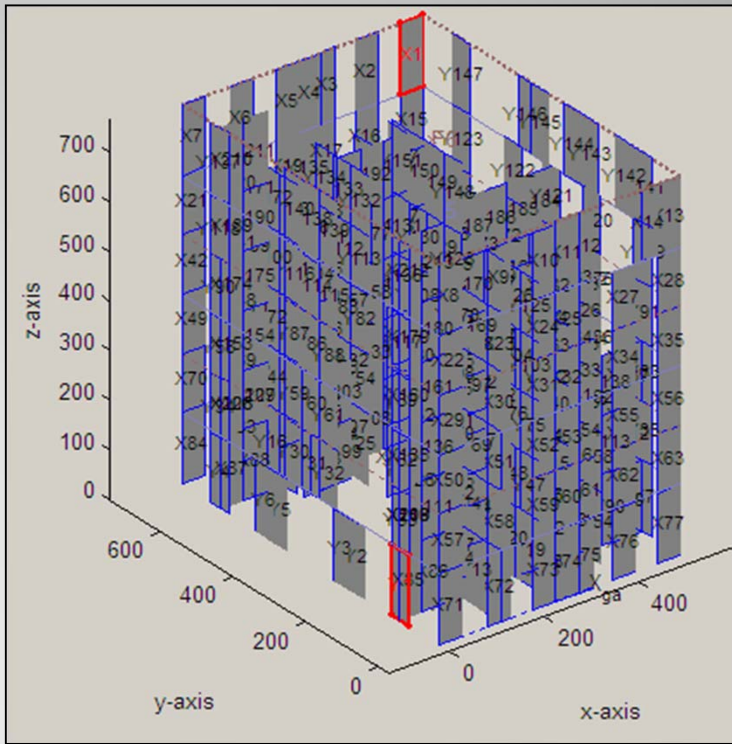
I. Numerical model 

- SAWS (MATLAB)
- Shearwalls modeled as hysteretic spring elements

II. Seismic hazard (OGM suites)

- 20 bi-axial records each
- Selected from the PEER database to match the design response spectra

III. Post-processing of results



- Extend a procedure for Direct Displacement-Based Design (DDD) of midrise wood frame (timber) buildings, *e.g.*, 3-6 stories

- Develop a set of (probability-based) factors for use in the DDD procedure to meet specified performance levels with certain target probabilities

- Create design charts (e.g., as a function of building height) to enable selection of appropriate C_{NE} factor for given target drift and non-exceedance probability

Toward Probabilistic DDD

(Pang and Rosowsky, 2009; Rosowsky and Yue, 2010)

1. Calculate vertical distribution factors for the base shear
2. Calculate normalized story shear factors
3. Calculate effective height
4. Calculate target displacement
5. Calculate effective seismic weight
6. Determine damping reduction factor
7. **Determine design base shear coefficient**
8. Calculate design forces (base shear, lateral forces, story shears)
9. Select shear walls

Simplified DDD procedure

Probabilistic DDD

- Original simplified procedure was median-based (50% non-exceedance)
- C_{NE} factors introduced as a way to design for non-exceedance probabilities *other than* 50% (increased flexibility in defining performance requirements)

Design base shear coefficient (C_c) for DDD procedure:

$$C_c = \min \begin{cases} \dfrac{C_{NE} S_{XS}}{B_\zeta} \\ \dfrac{g}{4\pi^2 \Delta_{eff}} \left(\dfrac{C_{NE} S_{X1}}{B_\zeta} \right)^2 \end{cases}$$

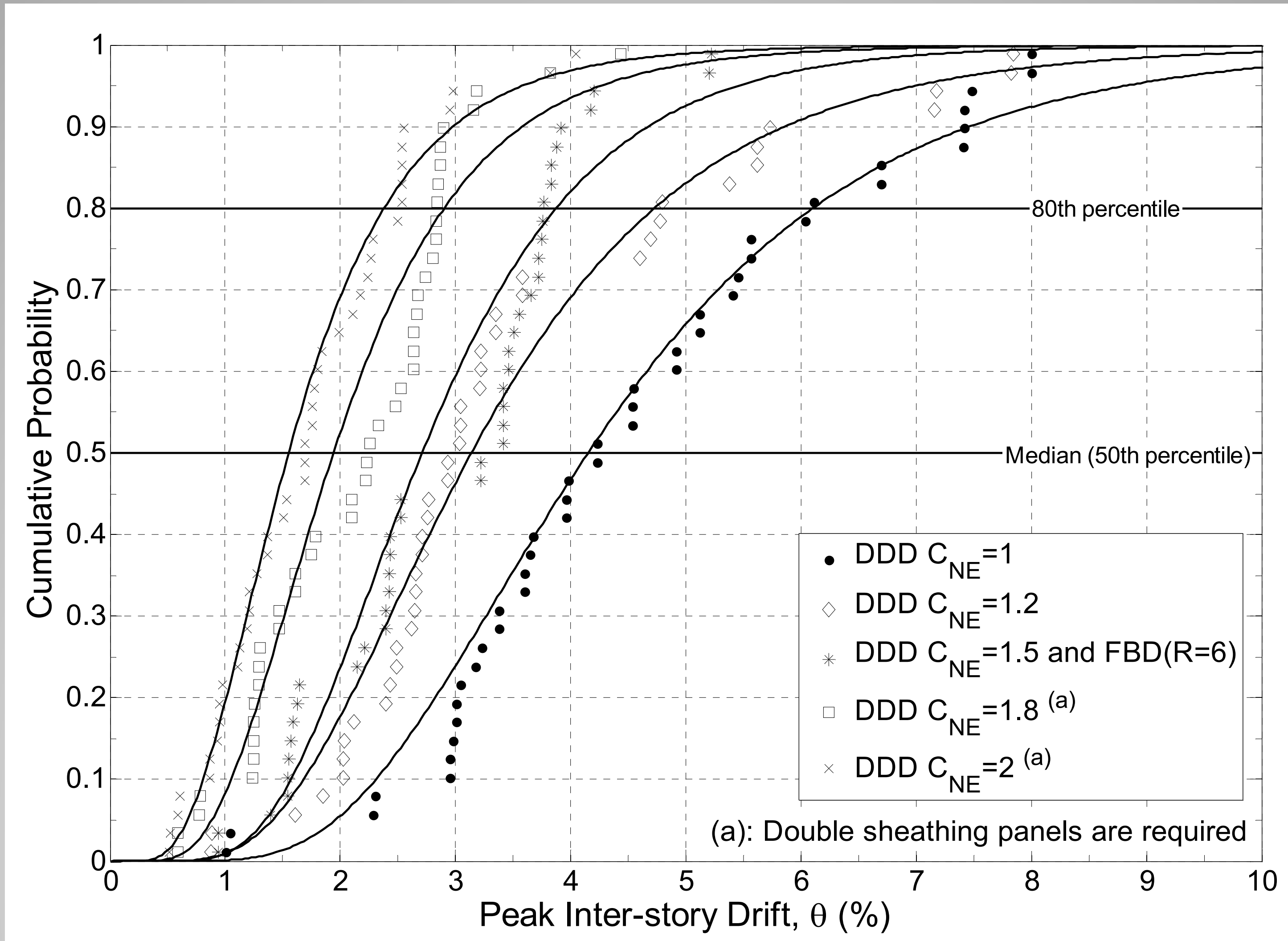
Vary factor from 1-2, re-design building, re-analyze (NLTHA) for drift profile using suite of ground motions, evaluate peak inter-story drift performance

where C_{NE} = adjustment factor for different $Pr(NE)$

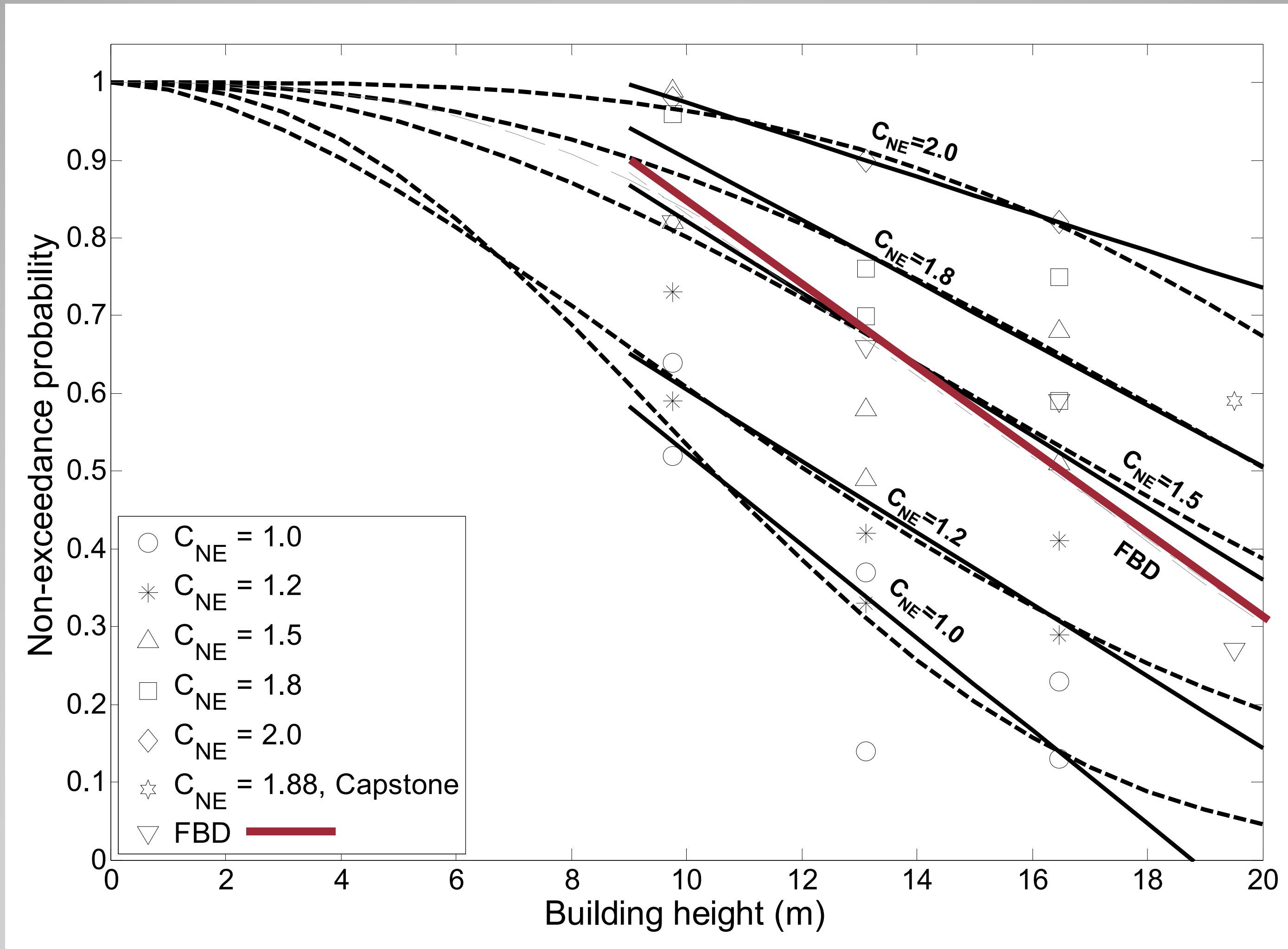
Base shear **demand** = product of the effective seismic weight and C_c

Shear wall **capacity** from database

Design: Total shear wall capacity > base shear demand



3-story ATC-63 archetype 10 structure under high seismic hazard (2%/50 yrs)



Performance-based design charts (C_{NE} factor)

Summary: first generation PB/DDD

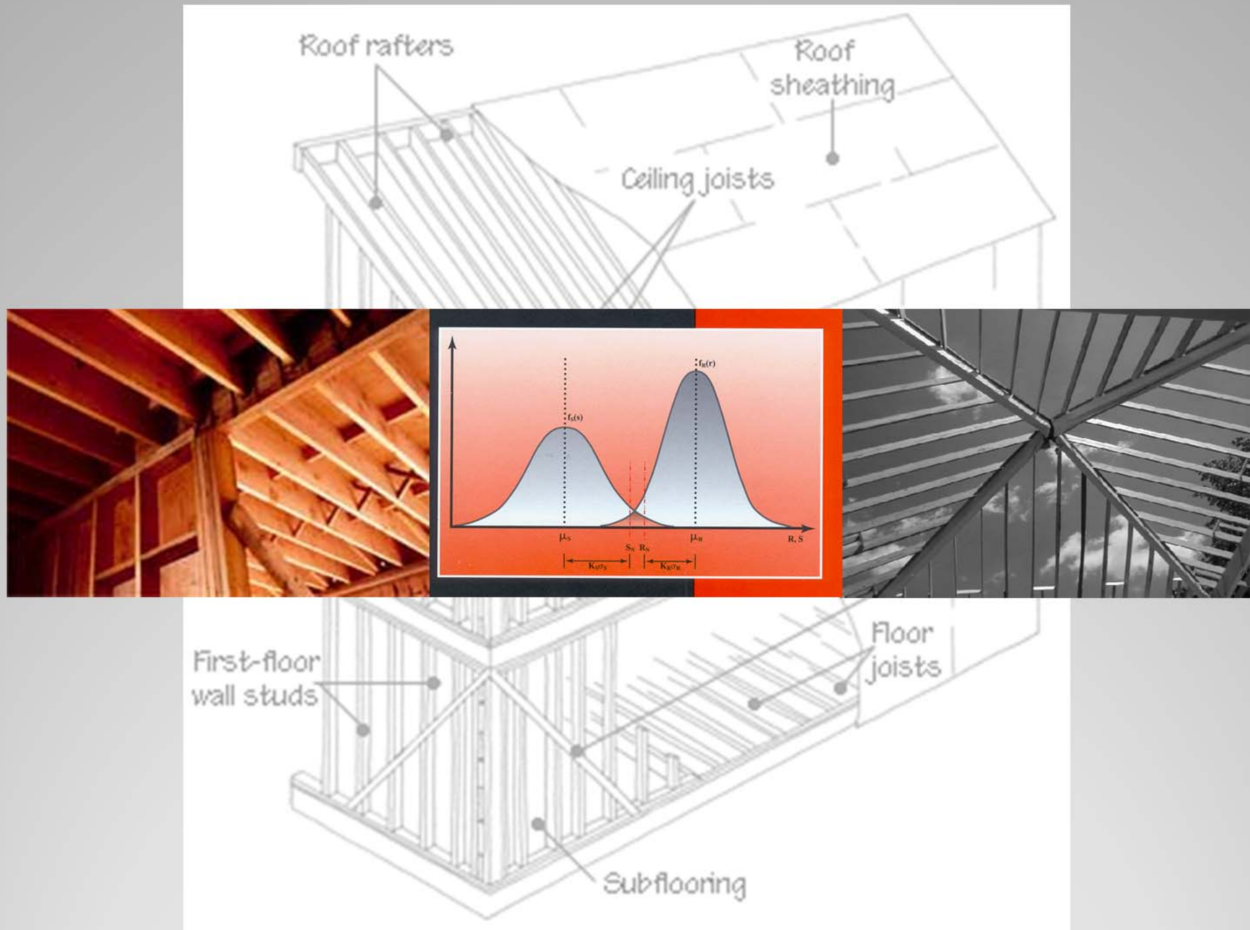
- The simplified DDD procedure has been extended into a risk-based PBD procedure through the introduction of C_{NE} factors, enabling the engineer to specify (1) target drift and (2) non-exceedance probability at a given hazard level
- Portfolio of archetype structures captures variability in building configurations; suite of scaled ground motion records captures variability in seismic hazard
- Proposed DDD procedure with C_{NE} factors is able to provide more risk-consistent designs across the range of building heights considered; this is advantageous in a PBD framework

- Reliability-based design concepts are now mature for timber structures, codes developed/maintained worldwide, partial factor format (member-based), region-specific design loads and material properties groupings
- Performance-based design concepts evolving worldwide, (first focus on seismic), multi-objective design, region-specific hazard characterization
- Fast and efficient MCS techniques have enabled time-dependent analyses, systems-level analyses, nonlinear time-history analyses, advanced modeling/simulation, complex structural-environmental interactions, etc.

In closing:
Evolution of probabilistic methods for timber structures

- Harmonization of LSD codes (across materials, countries)
- ASD 2.0 (where needed)
- Linkage between LSD and PBD (multi-tier, partial factors)
- Advances in whole structure modeling
- Multi-hazard design
- Performance-based design for durability (sustainability)

In closing:
What's next?


Roof rafters
Roof sheathing
Ceiling joists
First-floor wall studs
Floor joists
Subflooring

Thank you.