COST E55 – Zürich

Quantifying Ductility in Timber Structures

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Background and Motivation

EN 1990: Basis of design, Basic requirements 2.1

- (1): Sustain all actions
- (2): Adequate resistance, serviceability and durability
- (3): Fire resistance

- (4): A structure shall be designed and executed in such a way that it will not be damaged by events such as:
  - explosion,
  - impact, and
  - the consequences of human errors,

to an extent disproportinate to the original case.

Basis of design

- EN 1991-7: Accidental actions
  - Impact from traffic
  - Internal explosions

- EN 1995-1-1:
  - Only strength (and serviceability)

Basis of design

- EN 1998: Earthquakes
  - Static ductility (3 cycles, $0.8*F_u$):

$$D = \frac{\varepsilon_u}{\varepsilon_0}$$

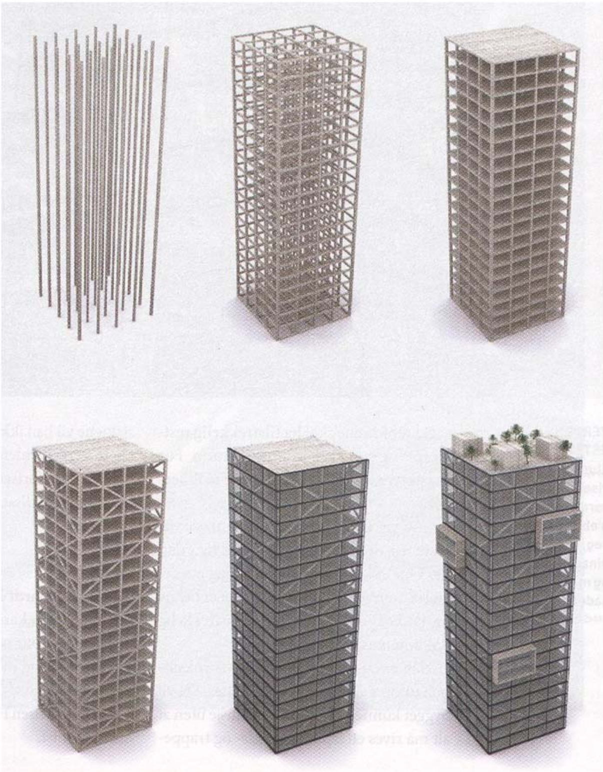
  - Ductility class:
    - Low (D< 4)
    - Medium (4<D<6)
    - High (D>6)
  - Take into account:
    - Initial slip in connections
    - E_0 values (+ 10%)

Single family housing or few stories

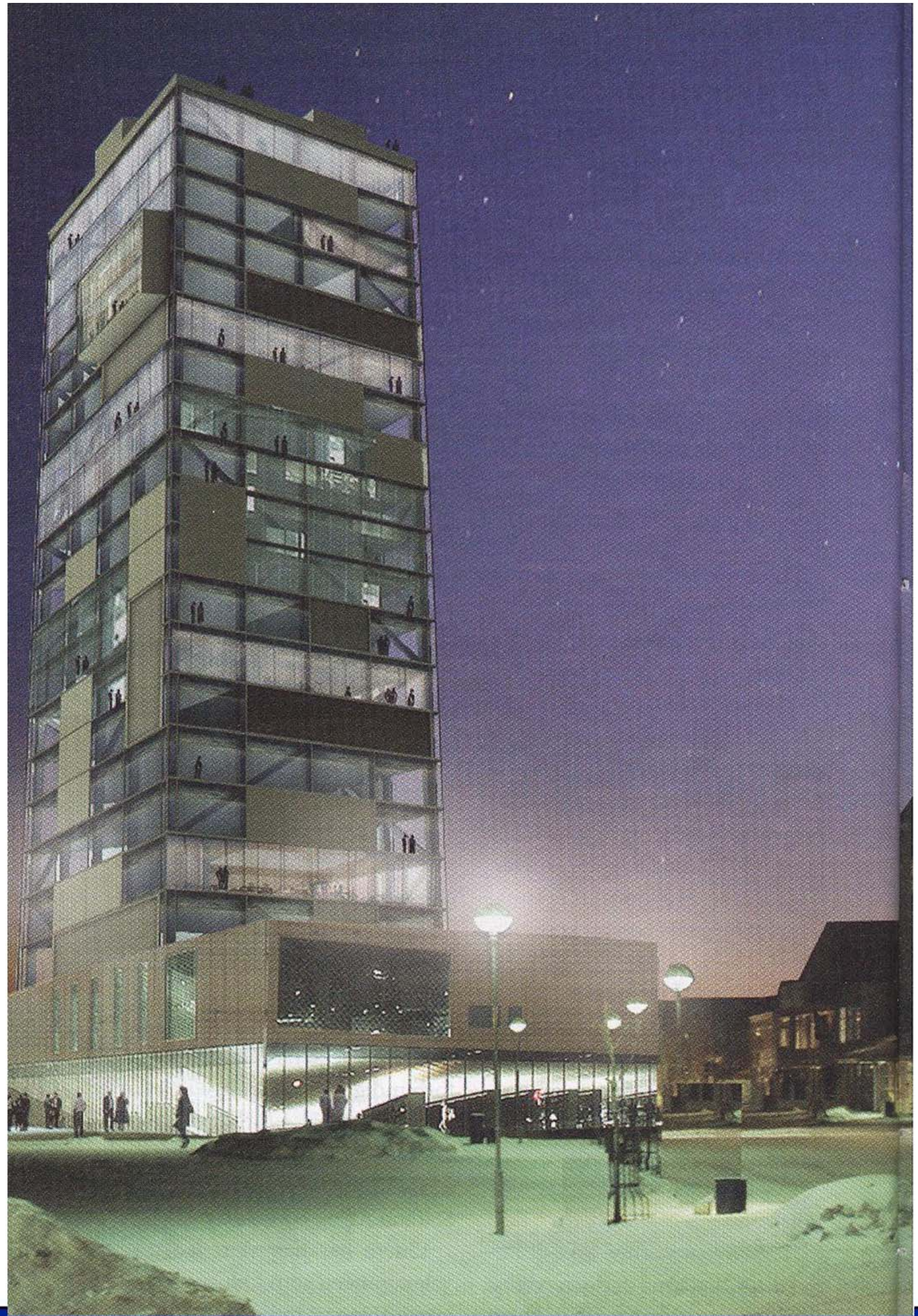
- Wooden structure housing damage at the 1995 Kobe earthquake (photo by Michio Miyano)

20 storied building?

(Ramstad arkitekter)



TRE: Skissene viser et alternativ til bærende konstruksjoner som vurderes i mulighetsstudien. Først 25 bærende søyler, deretter bjelkelag, 400 x500 mm før 180 mm massivtredekker legges inn. Så kommer diagonale kryssavstivere og til sist glassfasaden med utkragende bokser i massivtre. FOTO: REIULF RAMSTAD ARKITEKTER



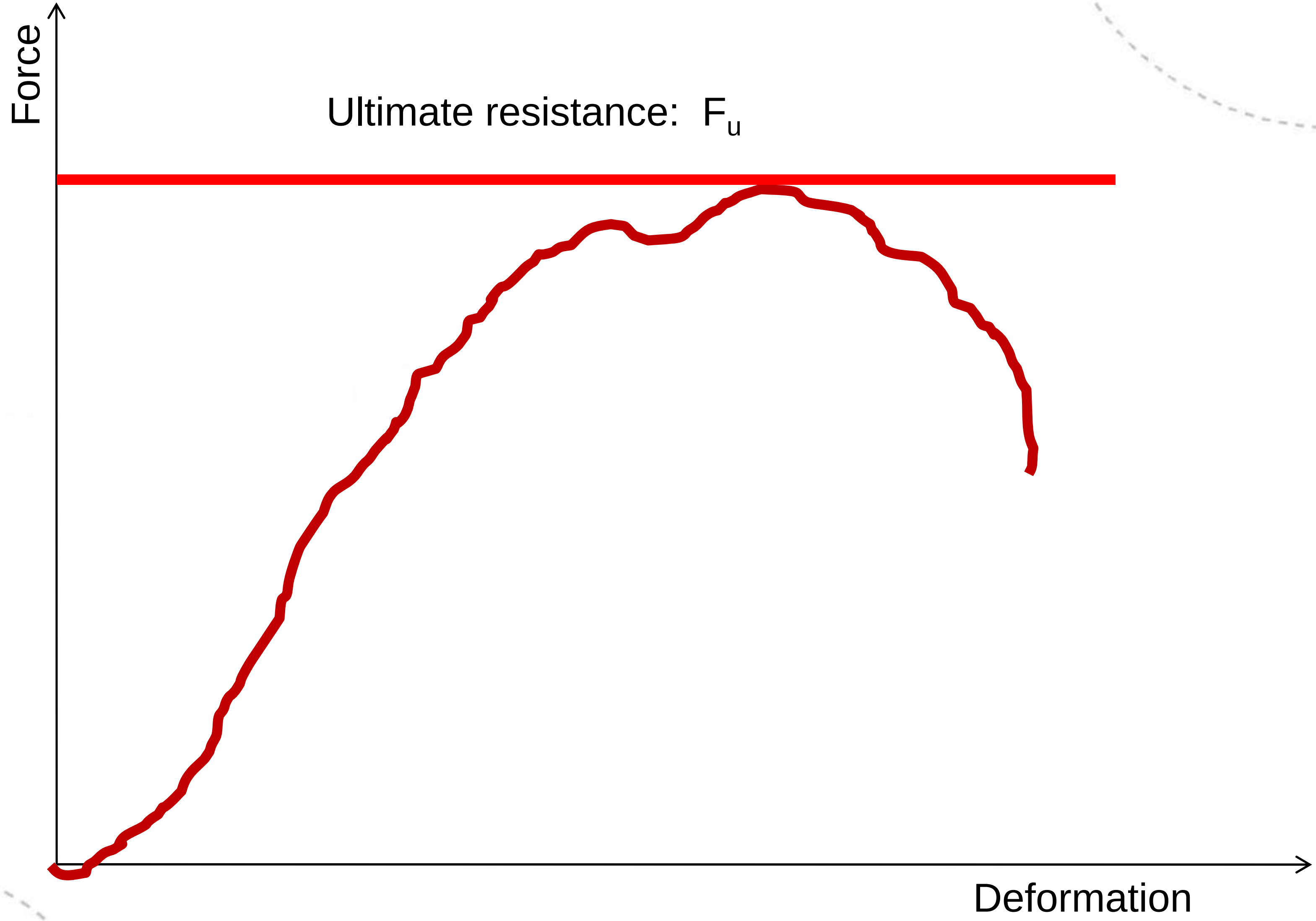
Loading Conditions
Testing
Failure modes

Loading and Failure Modes:

- Static loading (Load is constant)
  - Snow on roof
  - (Slowly) Increased variable load
  - Decrease in resistance

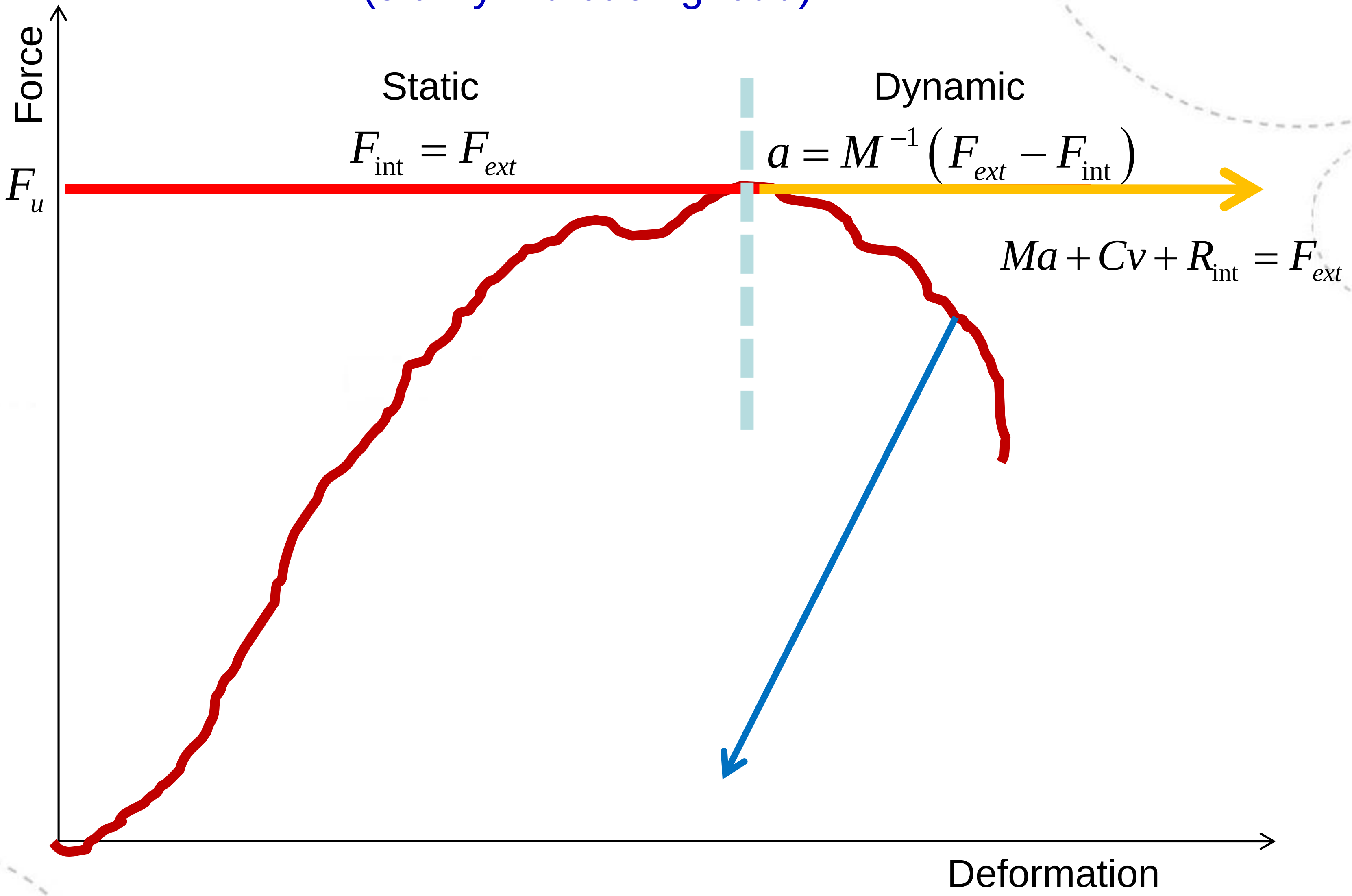
- Dynamic loading (Load time dependent)
  - Impact e.g. from wild vehicles
  - Impact from falling mass above
  - Explosion
  - Earthquake
  - Sudden change in load distribution due to partly failure (e.g. due to human errors?)

Deformation controlled test:

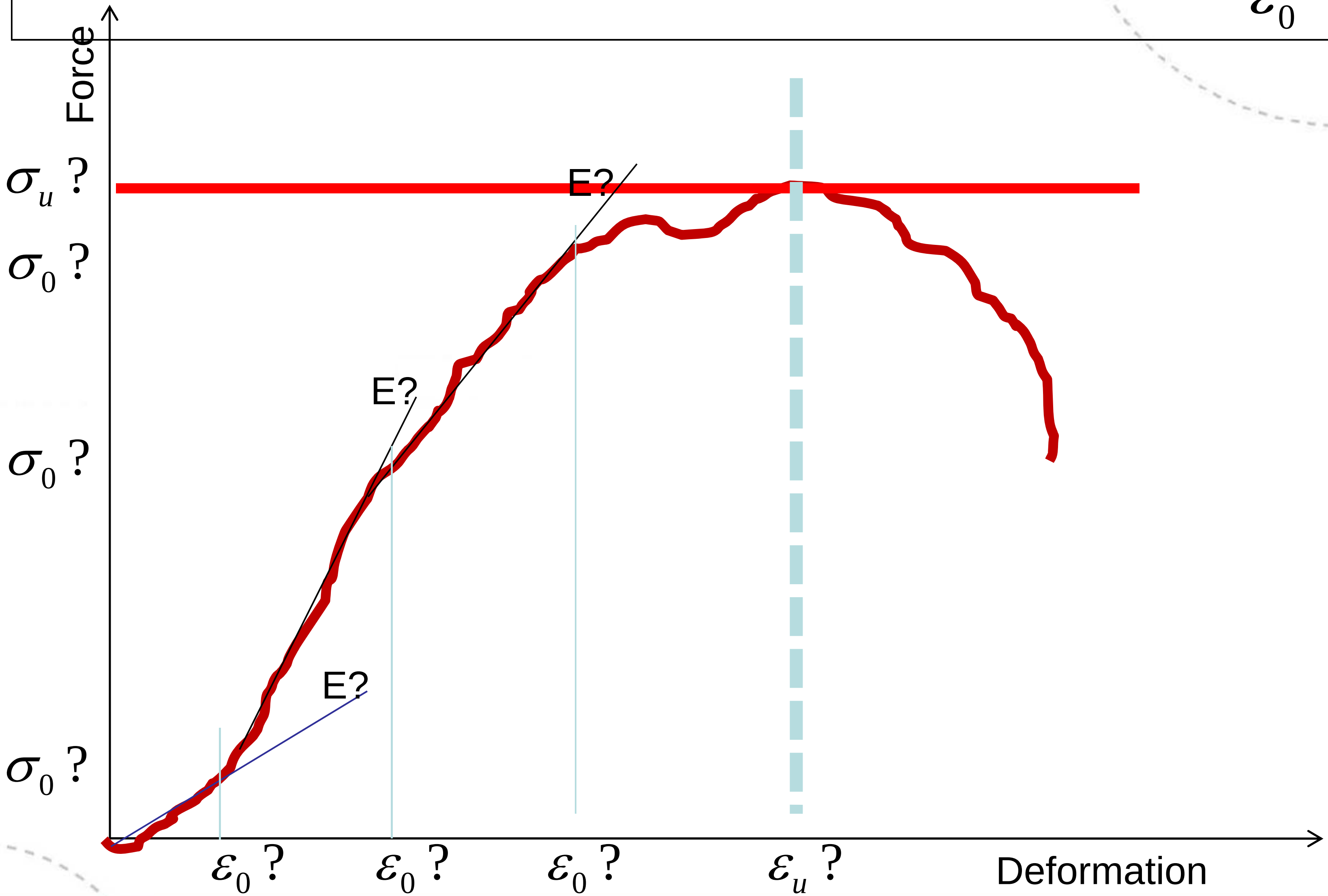


Load controlled test

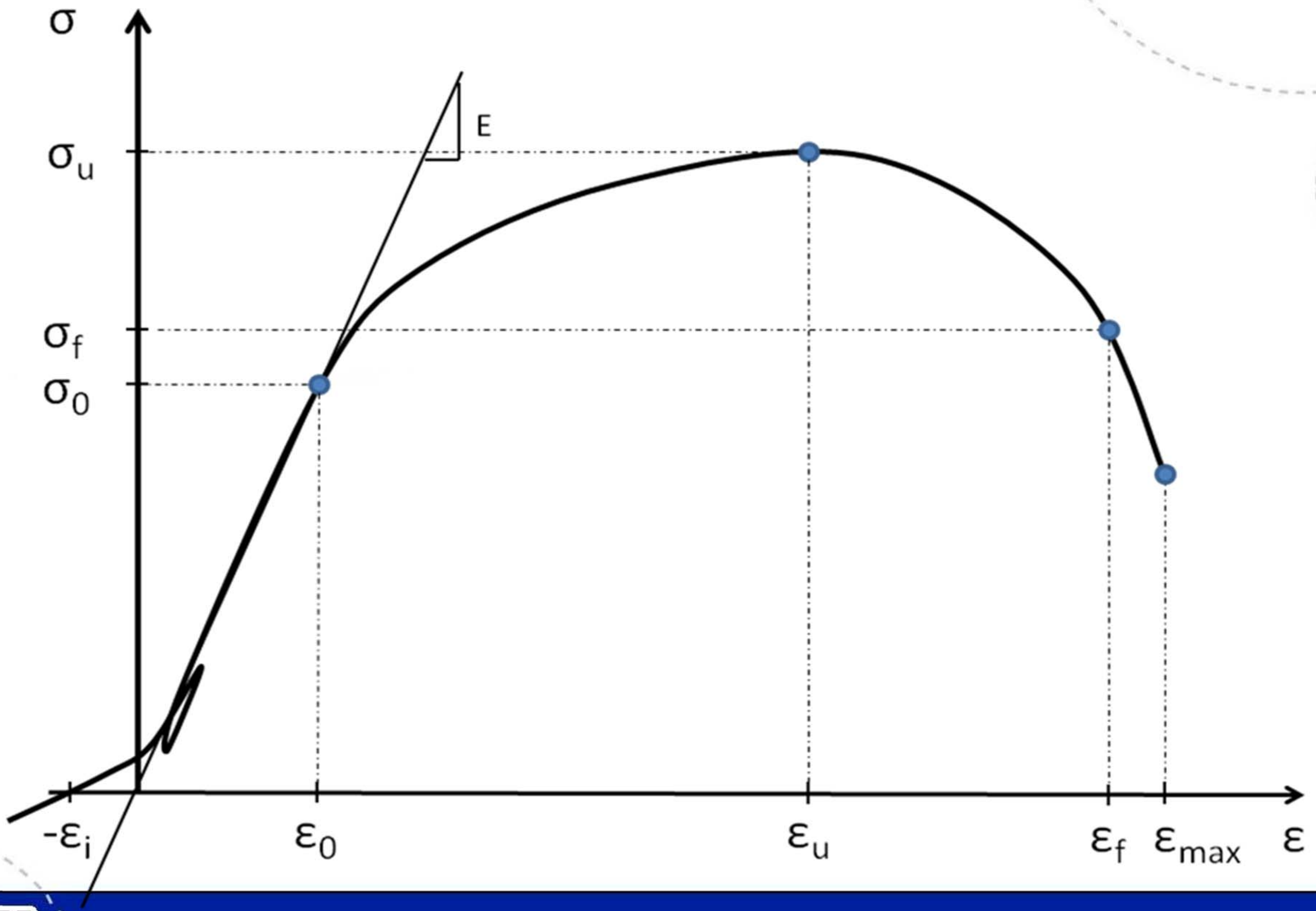
(slowly increasing load):



Parameters (EN 1998): *slip*, *E*, $D = \frac{\varepsilon_u}{\varepsilon_0}$?



Nomenclature:



Constitutive Modeling

Total strain models:

- In general: $\sigma = f(\varepsilon)$
- Power law: $\sigma = C_1 + C_2\varepsilon^n$
- Prager: $\sigma = \sigma_0 \tanh\left(E\varepsilon/\sigma_0\right)$
- Voce: $\sigma = C_3 + C_4\left(1-\exp\left(-C_5\varepsilon\right)\right)$
- Jorissen - Fragiacomo: $$\sigma = \sigma_0\left(1-\exp\left(-\frac{K_1}{\sigma_0}\varepsilon\right)\right) + K_1\varepsilon\left(1-\exp\left(-\frac{K_1}{\sigma_0}\varepsilon\right)\right) \leq \sigma_{\max}$$
- Menegotto Pinto: $$\frac{\sigma}{\sigma_0} = C_8\left(\frac{\varepsilon}{\varepsilon_0}\right) + \left(1-C_8\right)\frac{\frac{\varepsilon}{\varepsilon_0}}{\left[1+\left(\frac{\varepsilon}{\varepsilon_0}\right)^n\right]^{\frac{1}{n}}}$$

Constitutive piecewise models:

- Elastic - elastoplastic domain (piecewise - constant shift)

$$\sigma = \begin{cases} E\varepsilon & \text{for } 0 \le \varepsilon < \dfrac{\sigma_0}{E} = \varepsilon_0 \\ \sigma_0 + f(\varepsilon - \varepsilon_0) & \text{for } \varepsilon \ge \varepsilon_0 \end{cases}$$

- Decomposition of elastic and plastic deformation

$$\varepsilon = \varepsilon_e + \varepsilon_p$$

- Elastic strain $\varepsilon_e = \sigma / E$ Plastic strain $\varepsilon_p = \varepsilon - \dfrac{\sigma}{E}$

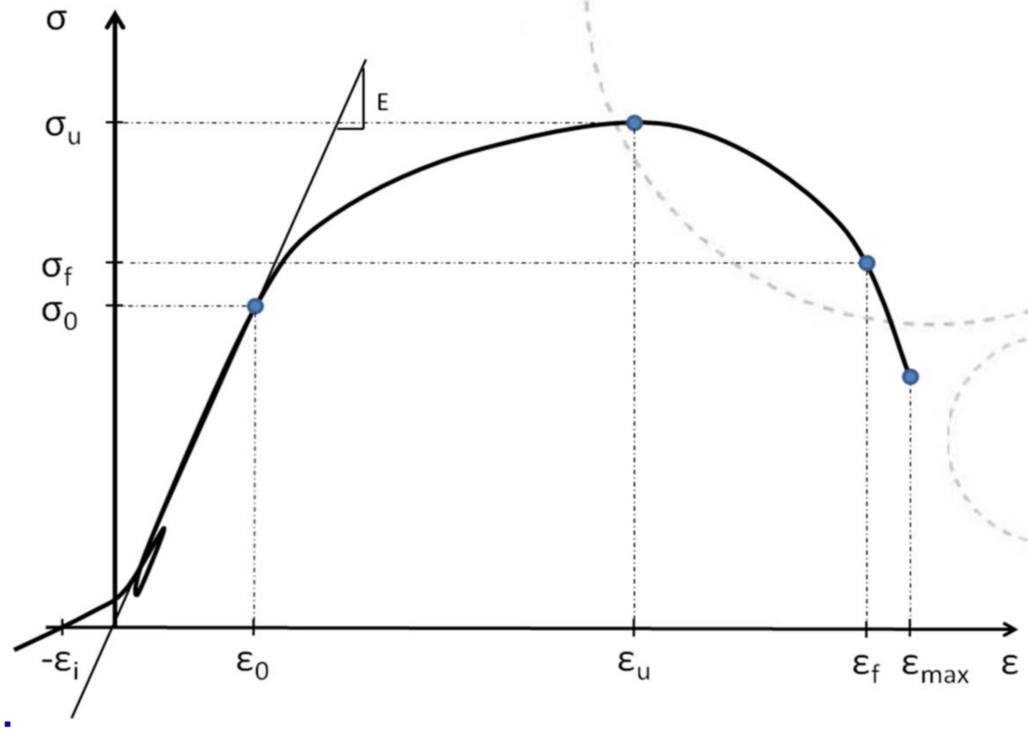
- Elastic or plastic domain (decoupled)

$$\sigma = \begin{cases} E \cdot \varepsilon & \varepsilon < \sigma_0 / E \\ f(\varepsilon_p) & \varepsilon \ge \sigma_0 / E \end{cases}$$

Plastic domain

- Stress-plastic strain; ≠

$$\sigma = f(\varepsilon_p)$$



- Maximum (ultimate) strain:

$$\frac{d\sigma}{d\varepsilon_p} = \frac{f(\varepsilon_p)}{d\varepsilon_p} = 0 \quad \rightarrow \varepsilon_{pua}$$

$$\varepsilon_{pu} = \min \begin{cases} \varepsilon_{p\max} \\ \varepsilon_{pua} \end{cases}$$

Fracture (softening branch)

- Total internal energy

$$W_t = W_e + W_p$$

$$W_e = \int_0^{\sigma_1} \sigma d\varepsilon_e = \frac{\sigma_1^2}{2E}$$

$$W_p = \int_0^{\varepsilon_{p1}} \sigma d\varepsilon_p = \int_0^{\varepsilon_{p1}} f(\varepsilon_p) d\varepsilon_p$$

$$\sigma = f(\varepsilon_p)$$

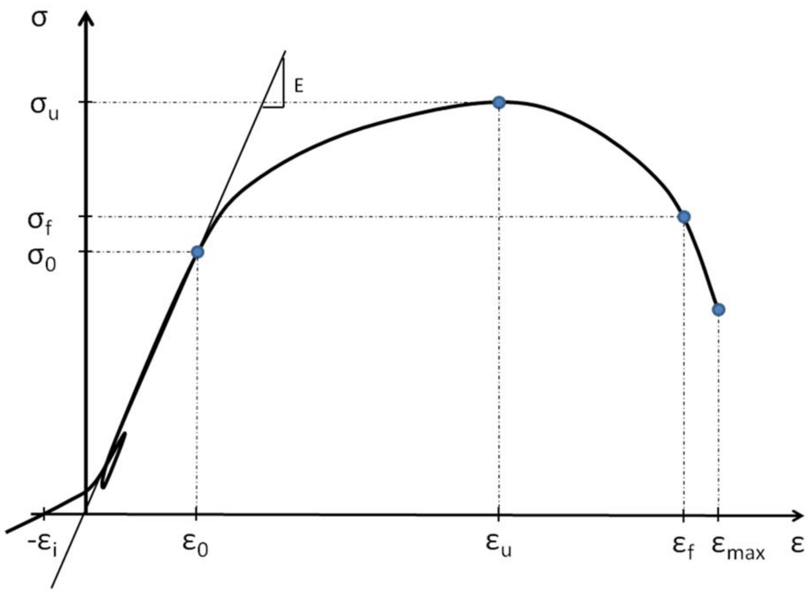
- Analytical fracture:

$$\frac{dW_t}{d\varepsilon_p} = \frac{dW_e}{d\varepsilon_p} + \frac{dW_p}{d\varepsilon_p} = \frac{\sigma}{E}\frac{d\sigma}{d\varepsilon_p} + \sigma = 0$$

$$\rightarrow \quad \frac{d\sigma}{d\varepsilon_p} = -E$$

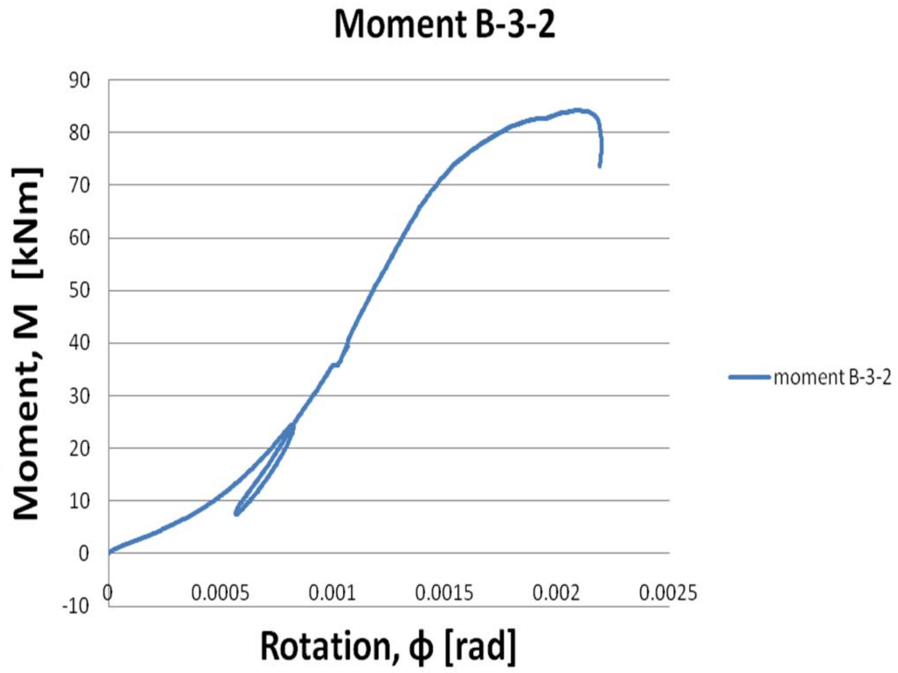
$$\rightarrow \quad \varepsilon_{pfa}$$

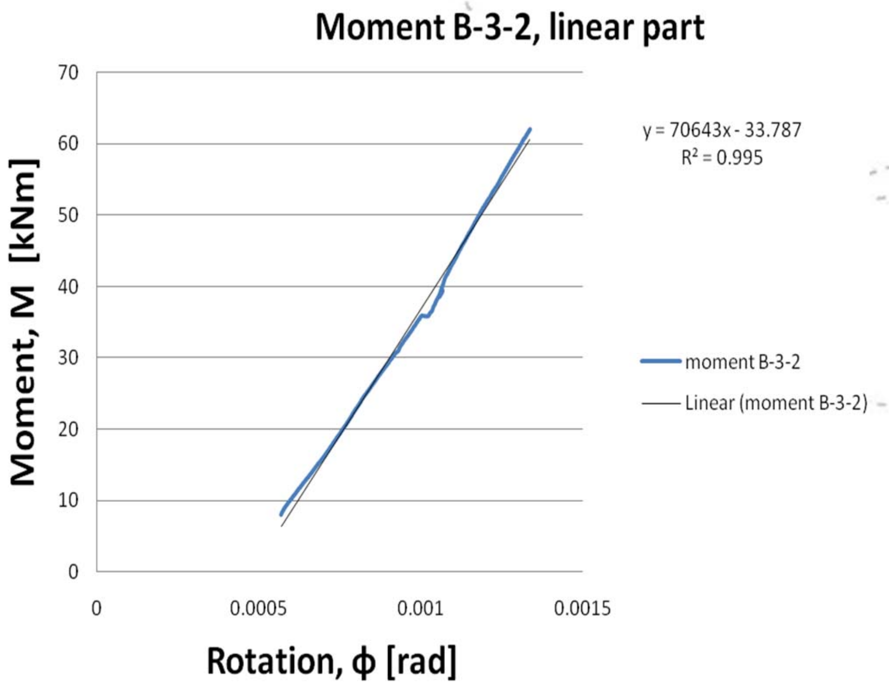
$$\varepsilon_{pf} = \min \begin{cases} \varepsilon_{p\max} \\ \varepsilon_{pfa} \end{cases}$$



Example 1: Moment vs. rotation

Linear Elastic model:





- Initial stiffness and slip (by regression) :

$$E = 70643 \quad \text{kNm/rad} \qquad \varepsilon_i\ (=\phi_i) = \frac{33.787}{70643} = 0.000478 \quad \text{rad}$$

- Linear elastic model: zero stress for zero strain

"Plastic" model

- Remove slip and elastic deformation

$$\varepsilon_p = \varepsilon - \varepsilon_i - \frac{\sigma}{E}$$

- Fit analytical expression (polynom?)

$$\sigma = f(\varepsilon_p)$$

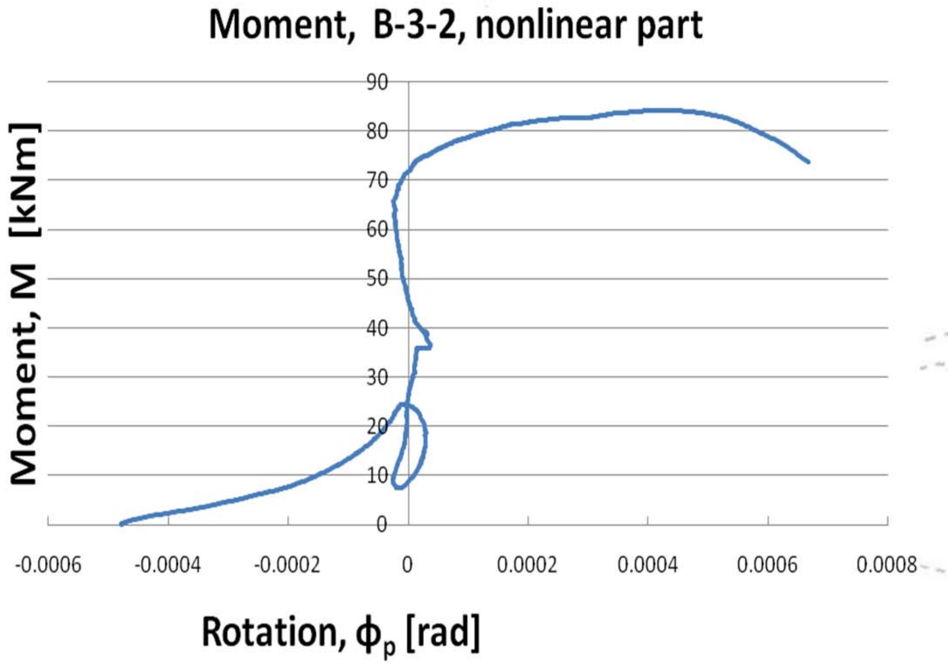
$$\sigma = \sigma_0 + A\varepsilon_p + B\varepsilon_p^2 + C\varepsilon_p^3$$

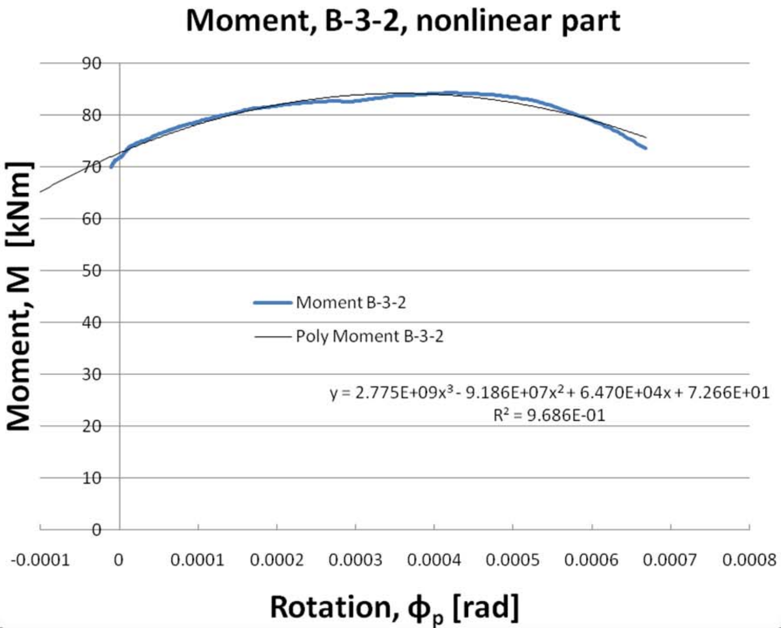
- Results:

$$\sigma_0\ (= M_0) = 72.663 \quad \text{kNm}$$
$$A = 64702 \quad \text{kNm/rad}$$
$$B = \text{-}9.19\cdot 10^7 \quad \text{kNm/rad}^2$$
$$C = 2.78\cdot 10^9 \quad \text{kNm/rad}^3$$

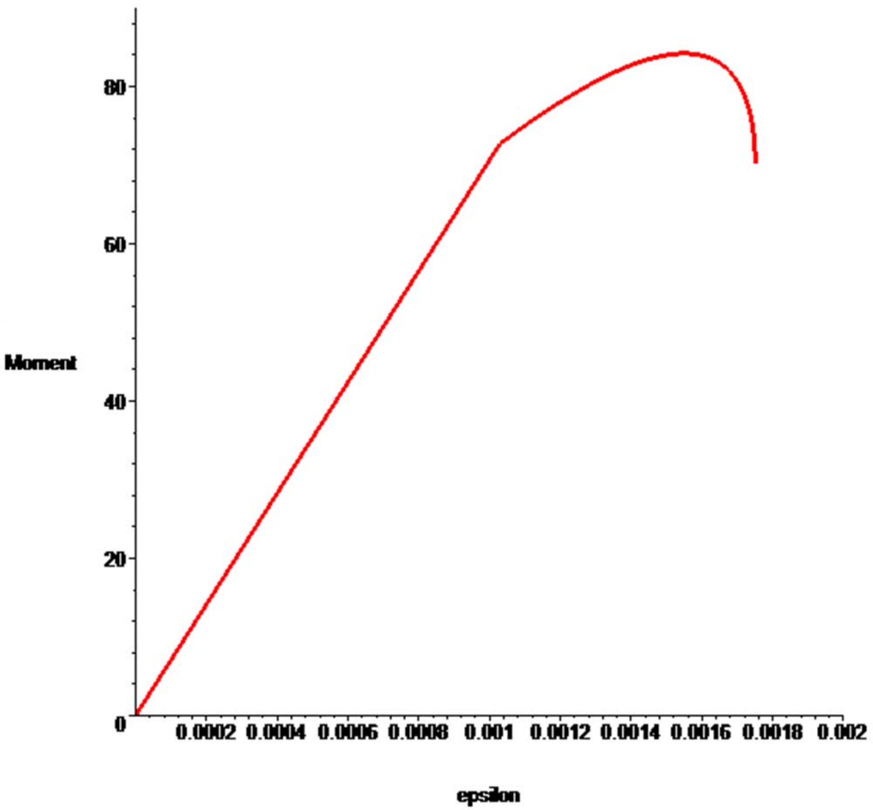
$$\varepsilon_{pu} = \min \begin{cases} \varepsilon_{p\max} = (0.00067) \\ \varepsilon_{pua} = 0.00036 \end{cases}$$

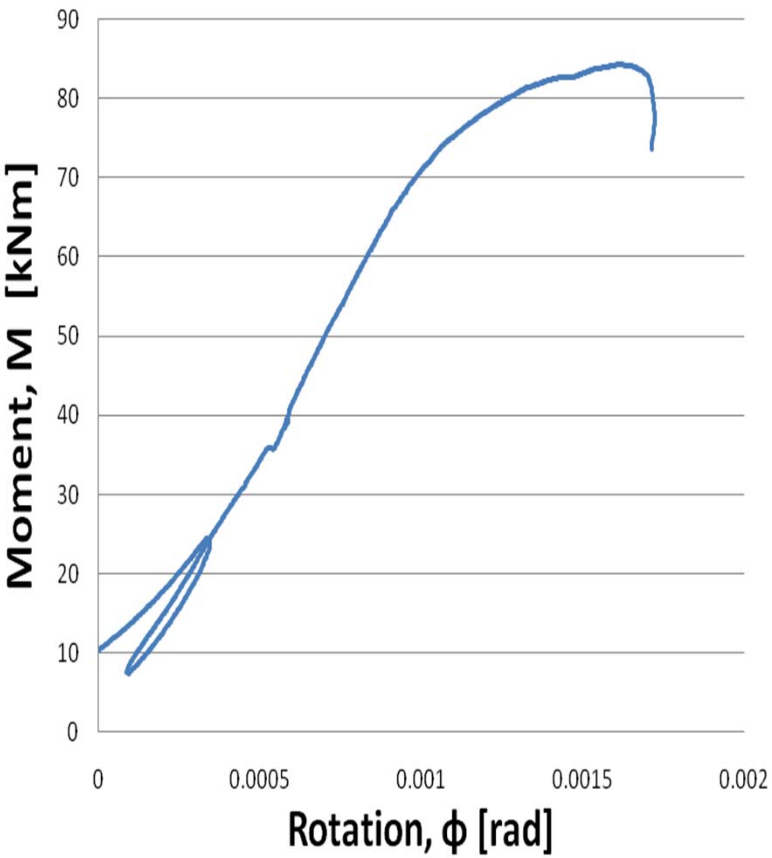
$$\varepsilon_{pf} = \min \begin{cases} \varepsilon_{p\max} = 0.00067 \\ \varepsilon_{pfa} = (0.00078) \end{cases}$$





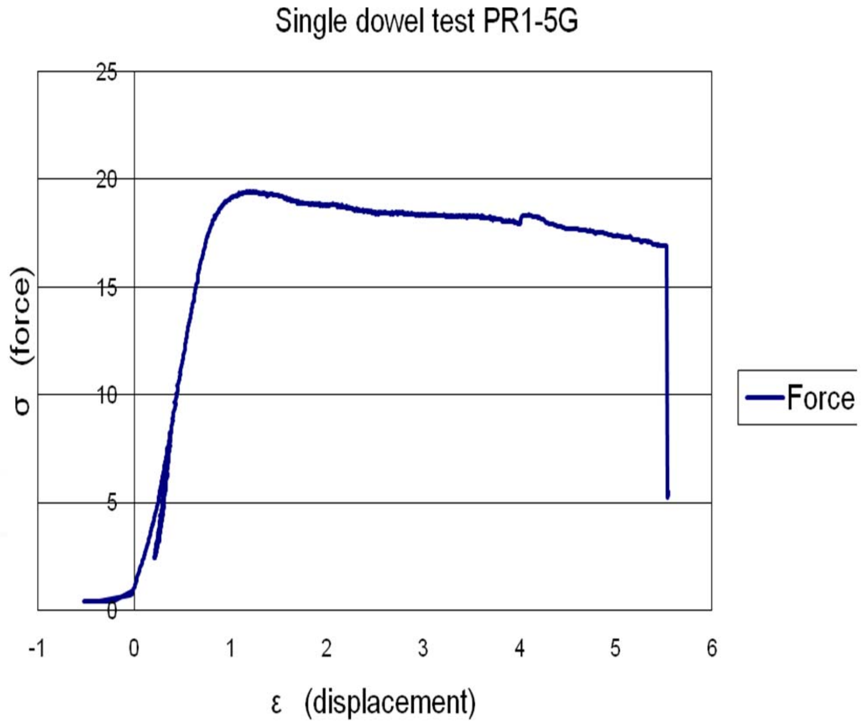
Model vs. Experiment (incl. slip)

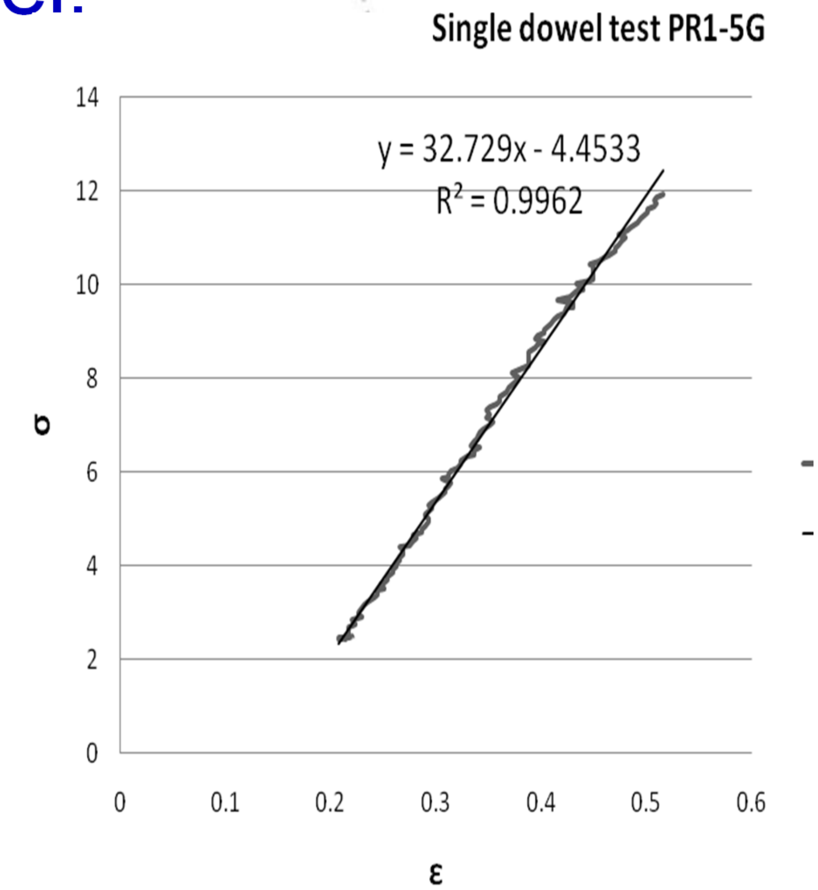




Example 2: Single dowel test

Linear Elastic model:





- Initial slip and stiffness:

$$E = 32.73 \quad \text{kN/mm} \qquad \varepsilon_i = \frac{4.4533}{32.729} = 0.136 \quad \text{mm}$$

"Plastic" model

- Remove slip and elastic deformation

$$\varepsilon_p = \varepsilon - \varepsilon_i - \frac{\sigma}{E}$$

- Fit analytical expression (2 terms Voce)

$$\sigma = f(\varepsilon_p)$$

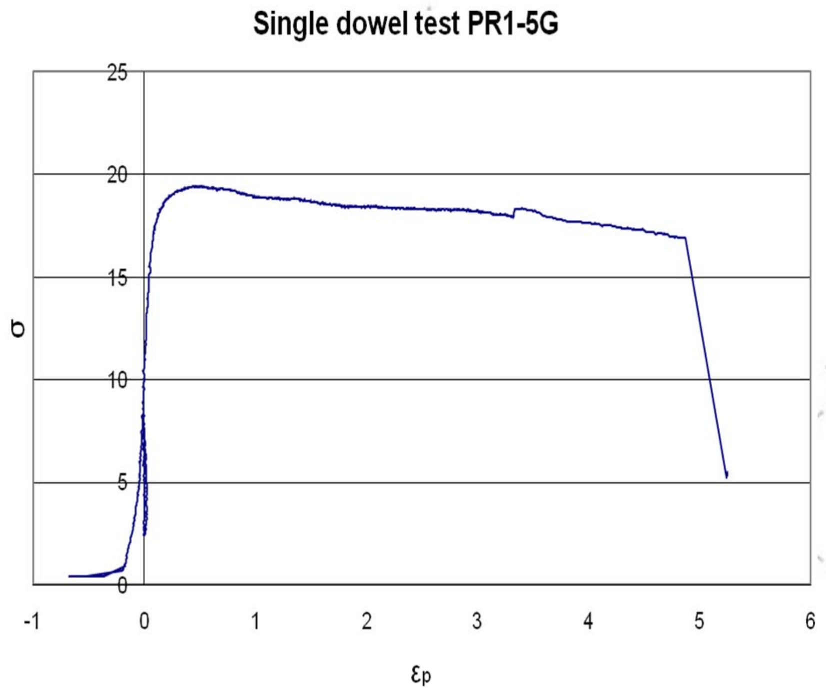
$$\sigma = \sigma_0 + Q_1\left(1 - e^{-C_1\varepsilon_p}\right) + Q_2\left(1 - e^{-C_2\varepsilon_p}\right)$$

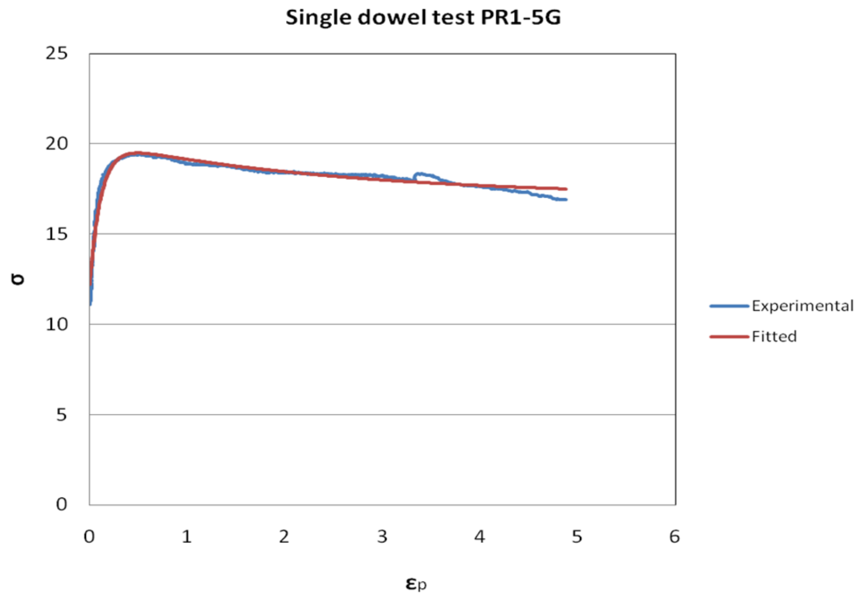
- Results

$$\sigma_0 = 12.14 \quad \text{kN}$$
$$Q_1 = 8.01 \quad \text{kN}$$
$$C_1 = 8.55 \quad \text{mm}^{-1}$$
$$Q_2 = -3.10 \quad \text{kN}$$
$$C_2 = 0.40 \quad \text{mm}^{-1}$$

$$\varepsilon_{pu} = \min\begin{cases}\varepsilon_{p\max} = (4.87)\\ \varepsilon_{pua} = 0.49\end{cases}$$

$$\varepsilon_{pf} = \min\begin{cases}\varepsilon_{p\max} = 0.00067\\ \varepsilon_{pfa} = (N.A.)\end{cases}$$

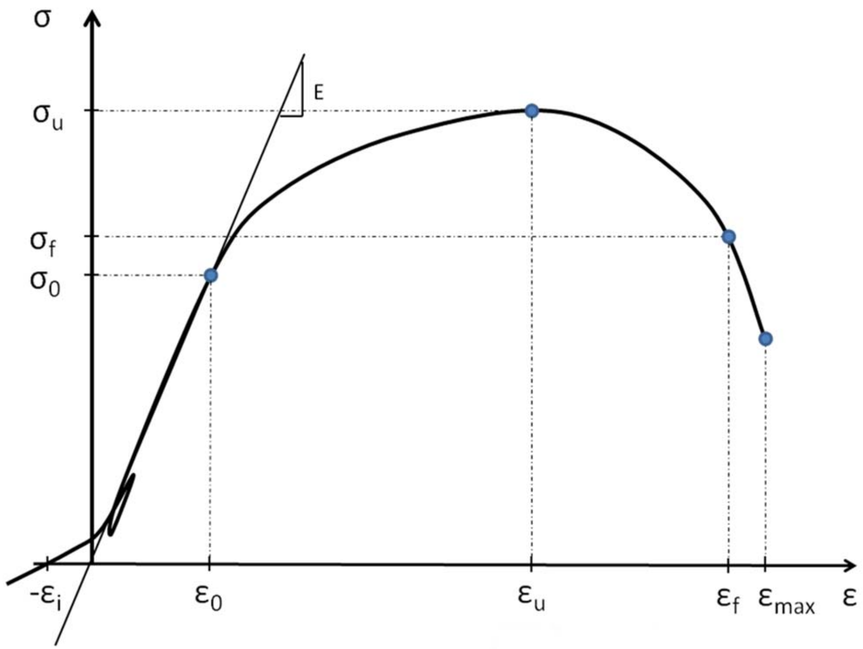


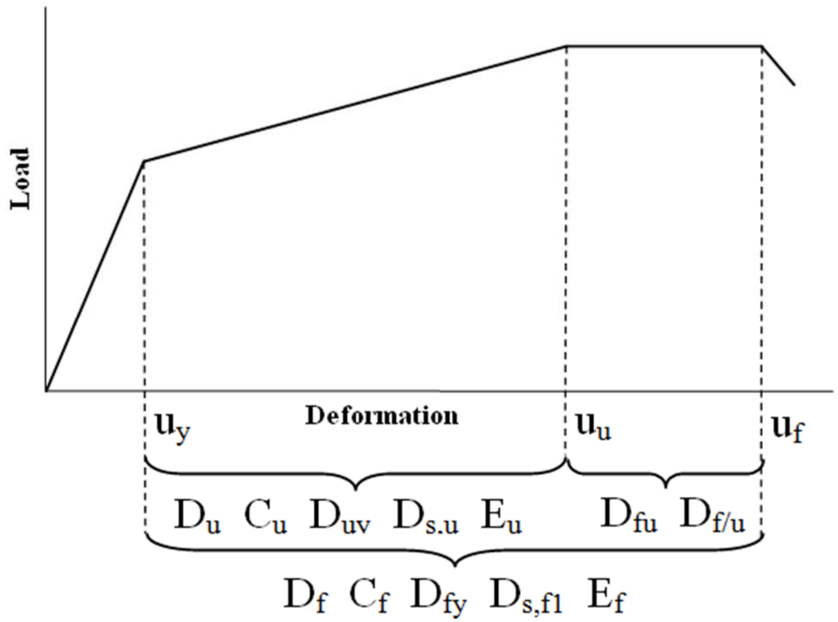


Ductility quantification

Ductility measures:

$$u_y = u_0 \rightarrow \varepsilon_0$$





Stehn-Bjørnfot, WCTE 2002:

$$D_f = \frac{u_f}{u_y} \quad (2) \qquad D_u = \frac{u_u}{u_y} \quad (3)$$

$$C_d = \frac{u_F - u_1}{u_F} \quad (4) \qquad D_{f/u} = \frac{u_f}{u_u} \quad (5)$$

$$D_{s,u} = \frac{K_1}{F_1} \cdot u_u \quad (6) \qquad D_{s,f1} = \frac{K_1}{F_1} \cdot u_f \quad (7)$$

$$D_{uy} = u_u - u_y \quad (8) \qquad D_{fy} = u_f - u_y \quad (9)$$

$$D_{fu} = u_f - u_u \quad (10) \qquad E_d = \int_0^{u_t} f(F,u)\,du \quad (11)$$

Strain based ductility measures "*Ds*"

$$Ds_{tu0} = \frac{\varepsilon_u}{\varepsilon_0} = \frac{\varepsilon_u}{\sigma_0 / E}$$

- $\varepsilon_u = \varepsilon_0$ Brittle material has ductility = 1 !! (EN 1998)

$$Ds_{pu0} = \frac{\varepsilon_{pu}}{\varepsilon_0} = \frac{\varepsilon_{pu}}{\sigma_0 / E}$$

$$Ds_{pf0} = \frac{\varepsilon_{pf}}{\sigma_0 / E}$$

- The offset from linearity is far from unique
  Scaling by "yield point" should be avoided

$$Ds_{tu} = \frac{\varepsilon_{pu}}{\varepsilon_u} = \frac{\varepsilon_{pu}}{\sigma_u / E + \varepsilon_{pu}}$$

$$Ds_{tf} = \frac{\varepsilon_{pf}}{\varepsilon_f} = \frac{\varepsilon_{pf}}{\sigma_f / E + \varepsilon_{pf}}$$

- Scaling by max. strain give 0 < D < 1, but insufficient distinctions between responses

$$Ds_{ue} = \frac{\varepsilon_{pu}}{\varepsilon_{eu}} = \frac{\varepsilon_{pu}}{\sigma_u / E}$$

- Scaling by elastic strain at ultimate force: OK.
  Static ductility up to ultimate force level.

$$\varepsilon_u = \frac{\sigma_u}{E}(1 + Ds_{ue})$$

- Easy strain calculation.

Energy based ductility measures "*Dw*"

$$Dw_{ue} = \frac{W_{pu}}{W_{eu}} = \frac{\int_0^{\varepsilon_{pu}} \sigma d\varepsilon_p}{\int_0^{\sigma_u} \sigma d\varepsilon_e} = \frac{\int_0^{\varepsilon_{pu}} f(\varepsilon_p) d\varepsilon_p}{\frac{\sigma_u^2}{2E}}$$

- For static loading (up to ultimate load)
  Plastic energy dissipation/elastic energy

$$Dw_{fe} = \frac{W_{pf}}{W_{eu}} = \frac{\int_0^{\varepsilon_{pf}} \sigma d\varepsilon_p}{\int_0^{\sigma_u} \sigma d\varepsilon_e} = \frac{\int_0^{\varepsilon_{pf}} f(\varepsilon_p) d\varepsilon_p}{\frac{\sigma_u^2}{2E}}$$

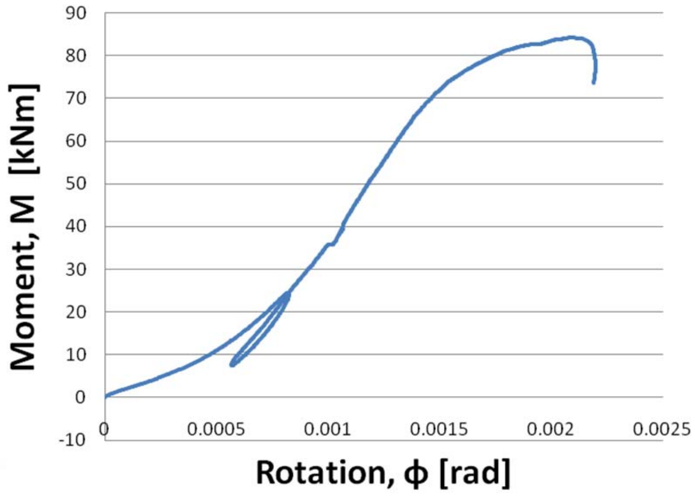
- Dynamic loading (up to fracture)
  Plastic dissipation/max elastic energy

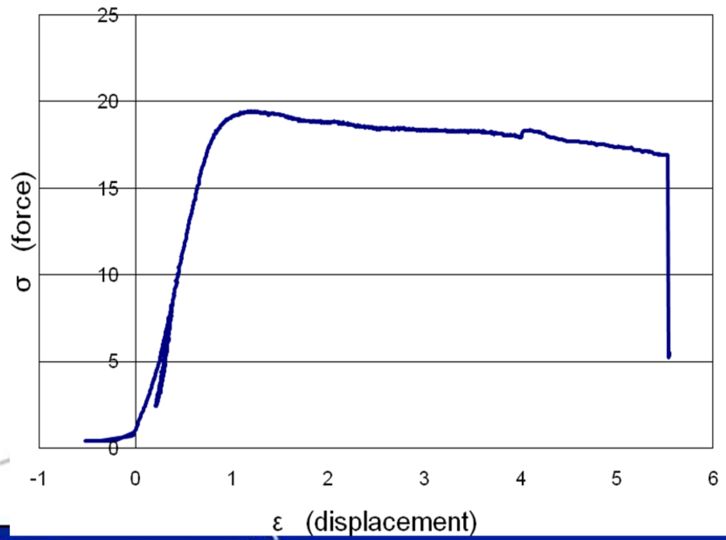
$$W_{tu} = \frac{\sigma_u^2}{2E}\left(1 + Dw_{ue}\right)$$

$$W_{tf} = \frac{\sigma_u^2}{2E}\left(\left(\frac{\sigma_f}{\sigma_u}\right)^2 + Dw_{fe}\right)$$

- Total energies

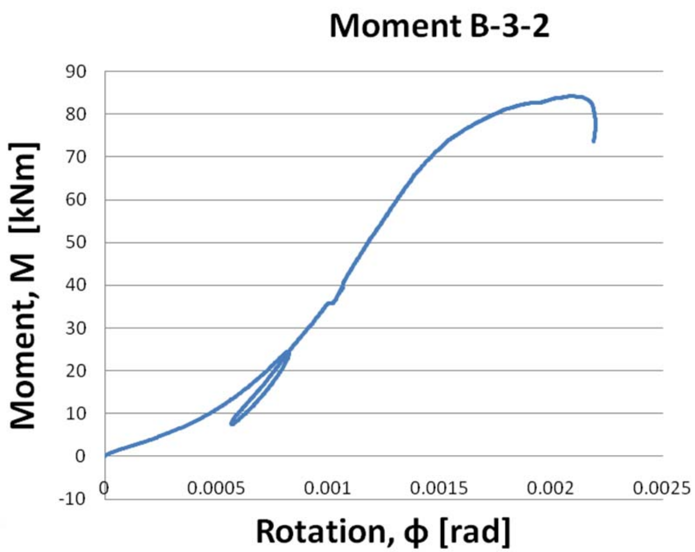
2 examples:

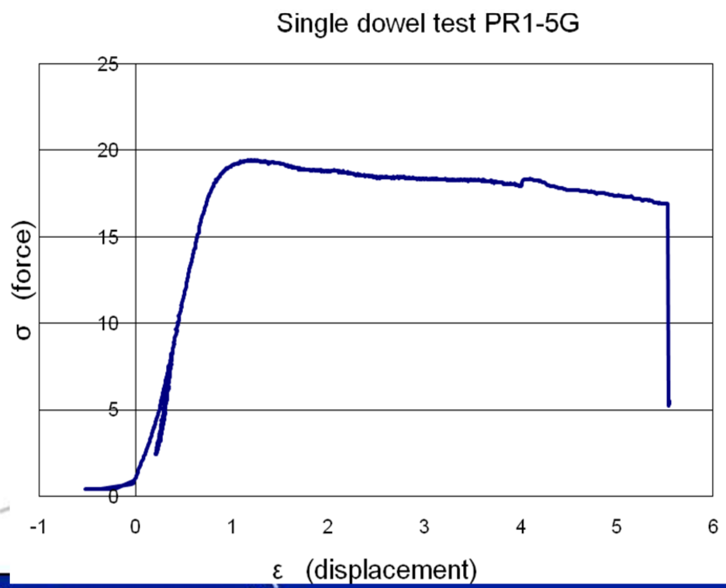




Parameters	*Moment Connection* [kNm and radians]	*Dowel test* [kN and mm]
E	70643	32.73
σ_0	72.7	12.14
σ_u	84.2	19.48
ε_i	0.000478	0.136
ε_{pu}	0.000358	0.492
ε_{pf}	0.000669	4.87
$Ds_{ue} = \frac{\varepsilon_{pu}}{\varepsilon_{eu}} = \frac{\varepsilon_{pu}}{\sigma_u / E}$	0.30	0.83
$Dw_{ue} = \frac{W_{pu}}{W_{eu}}$	0.57	1.53
$Dw_{ub} = \frac{W_{pu}}{\sigma_u \varepsilon_{pf}}$	0.45	0.09
$Dw_{fe} = \frac{W_{pf}}{W_{eu}}$	1.21	15.31
$Dw_{fb} = \frac{W_{pf}}{\sigma_u \varepsilon_{pf}}$	0.95	0.94

2 examples:





- Static loading:

$$Ds_{ue} = \frac{\varepsilon_{pu}}{\varepsilon_{eu}} = \frac{\varepsilon_{pu}}{\sigma_u / E} \rightarrow \frac{\text{Dowel test}}{\text{Moment test}} = \frac{0.83}{0.30} = 2.8$$

$$Dw_{ue} = \frac{W_{pu}}{W_{eu}} \rightarrow \frac{\text{Dowel test}}{\text{Moment test}} = \frac{1.53}{0.57} = 2.7$$

- Dynamic loading:

$$Dw_{fe} = \frac{W_{pf}}{W_{eu}} \rightarrow \frac{\text{Dowel test}}{\text{Moment test}} = \frac{15.31}{1.21} = 12.7$$

Final Remarks (Quantifying):

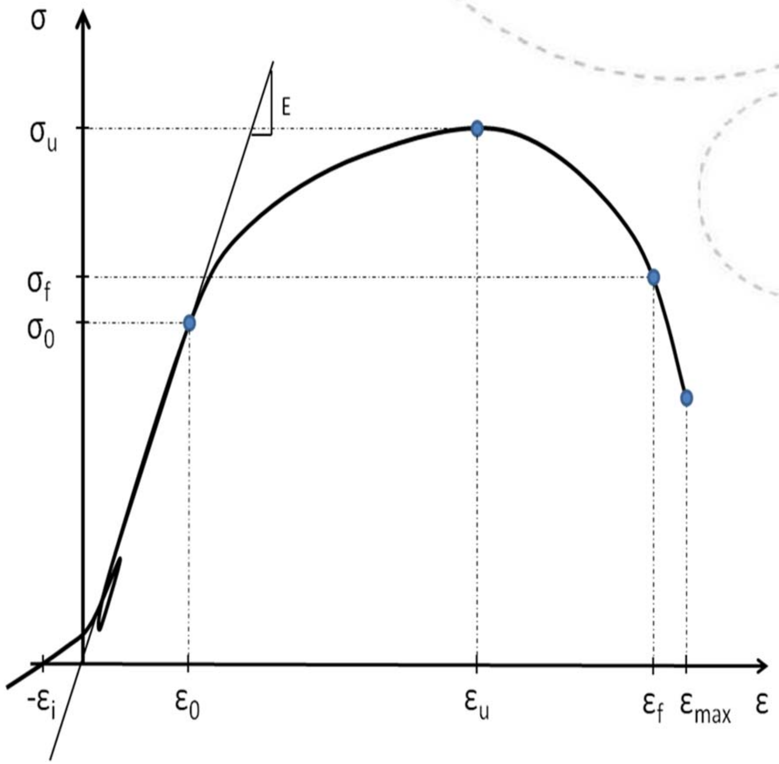
- Decomposition of strains:

$$\varepsilon = \varepsilon_e + \varepsilon_p \qquad \varepsilon_e = \sigma / E$$

- Decoupled models:

$$\sigma = \begin{cases} E \cdot \varepsilon & \varepsilon < \sigma_0 / E \\ f(\varepsilon_p) & \varepsilon \geq \sigma_0 / E \end{cases}$$

- Quantifying measured test response by regression on to analytical models
  - Initial slip
  - Elastic linear response
  - “Plastic” nonlinear response
- Determine parameters of the analytical models
- Compute derived properties from the analytical models



Final Remarks: Ductility

- Static loading:
  - Strain based:

$$Ds_{ue} = \frac{\varepsilon_{pu}}{\varepsilon_{eu}} = \frac{\varepsilon_{pu}}{\sigma_u / E}$$

  - Energy based:

$$Dw_{ue} = \frac{W_{pu}}{W_{eu}} = \frac{\int_0^{\varepsilon_{pu}} \sigma d\varepsilon_p}{\int_0^{\sigma_u} \sigma d\varepsilon_e} = \frac{\int_0^{\varepsilon_{pu}} f(\varepsilon_p) d\varepsilon_p}{\frac{\sigma_u^2}{2E}}$$

- Impact:

$$Dw_{fe} = \frac{W_{pf}}{W_{eu}} = \frac{\int_0^{\varepsilon_{pf}} \sigma d\varepsilon_p}{\int_0^{\sigma_u} \sigma d\varepsilon_e} = \frac{\int_0^{\varepsilon_{pf}} f(\varepsilon_p) d\varepsilon_p}{\frac{\sigma_u^2}{2E}}$$